International Comovement in the Global Production Network*

Zhen Huo
Yale University

Andrei A. Levchenko
University of Michigan
NBER and CEPR

Nitya Pandalai-Nayar
University of Texas at Austin
and NBER

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Abstract

This paper provides a general framework to study the role of production networks in international GDP comovement. We first derive an additive decomposition of bilateral GDP comovement into components capturing shock transmission and shock correlation. We quantify this decomposition in a parsimonious multi-country, multi-sector dynamic network propagation model, using data for the G7 countries over the period 1978-2007. Our main finding is that while the network transmission of shocks is quantitatively important, it accounts for a minority of observed comovement under the estimated range of structural elasticities. Contemporaneous responses to correlated shocks in the production network are more successful at generating comovement than intertemporal propagation through capital accumulation. Extensions with multiple shocks, nominal rigidities, and international financial integration leave our main result unchanged. A combination of TFP and labor supply shocks is quantitatively successful at reproducing the observed international business cycle.

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1. Introduction

Production networks play an important role in transmitting shocks within countries and amplifying aggregate fluctuations.[1] Since input networks increasingly cross national borders, a natural conjecture is that they may also help explain the positive correlation of real GDP growth across countries. Although there is a growing body of work on the international propagation of disaggregate shocks through interconnected sectors and firms (e.g. di Giovanni, Levchenko, and Mejean, 2018; Boehm, Flaaen, and Pandalai-Nayar, 2019), we still lack a comprehensive theoretical and quantitative account of the role of production networks in international GDP synchronization.

This paper develops and quantifies a general framework to study international comovement in the presence of global production networks. We first derive an additive decomposition of bilateral GDP comovement into components capturing shock transmission and shock correlation. We then set up a parsimonious and tractable multi-country, multi-sector dynamic network propagation model that can be implemented using widely available data. Using this framework, we quantify the relative importance of correlated shocks versus shock transmission in generating GDP comovement. In the model, shocks transmit internationally and generate positive comovement, consistent with micro evidence. However, the transmission of shocks through the input network accounts for a minority of observed comovement under the estimated values of structural elasticities. Fully reproducing observed comovement requires correlated shocks.

Section 2 sets up a simple accounting framework that clarifies the mechanisms at play and objects of interest for measurement. Two countries can experience positive comovement if shocks in one country influence the other country's GDP through trade and production linkages. Comovement also arises if influential sectors in the two economies have correlated shocks. The GDP correlation between two countries can be expressed as a function of the primitive shock covariances and a global influence matrix. The latter collects the general equilibrium elasticities of GDP in each country with respect to all sector-country-specific shocks worldwide, and thus translates the variances and covariances of the primitive shocks into comovements of GDP. We show that the GDP correlation between two countries can always be written as a sum of two terms, respectively capturing correlated shocks and transmission.

The accounting framework provides a roadmap for quantification and measurement. First, as the global influence matrix will always be model-dependent, we must impose sufficient structure and bring sufficient data on the international production network linkages to build the global influence matrix. Second, we must recover some underlying shocks to determine the extent of their correlation across countries. This will allow us to establish both how the influence matrix interacts with the shock correlation, and how it produces transmission.

[1]The closed-economy literature on the micro origins of aggregate fluctuations and on shock propagation via input linkages goes back to Long and Plosser (1983), and has been modernized recently following the seminal contribution by Acemoglu et al. (2012).

Section 3 sets up a dynamic multi-country, multi-sector, multi-factor model of the world economy. Countries trade both intermediate and final goods. Each sector uses labor, capital, and intermediate inputs that can come from any sector and country in the world. Sector-specific supply shocks propagate through the production network both domestically and internationally. In response to both domestic and foreign shocks, capital can be accumulated in each sector subject to a time-to-build lag. We provide a first-order analytical solution for the contemporaneous GDP impact of a global vector of shocks in the multi-country dynamic general equilibrium. This analytical solution expresses the response to shocks in terms of observables that can be measured and a set of structural elasticities.

Our baseline framework generalizes the canonical network propagation model (e.g. Acemoglu et al., 2012) in three dimensions. First, our solution applies in the international setting. We write the change in GDP of a single country as an inner product of the vector of shocks to all countries and sectors in the world and the country-specific influence vector that collects the elasticities of that country's GDP to every sectoral shock in the world. Second, the model features endogenous factor supply. This is important for analyzing business cycle transmission to measured GDP. In the canonical network model factor supply is fixed, and thus measured GDP by construction does not respond to foreign shocks. Third, our analytical solution for the contemporaneous change in GDP applies in a dynamic setting where shocks also propagate intertemporally. Our analysis thus integrates the static network literature that follows Acemoglu et al. (2012) and the dynamic international business cycle literature (e.g. Backus, Kehoe, and Kydland, 1992). We can cleanly separate the intratemporal propagation analyzed in the former and the delayed responses to shocks emphasized by the latter.

Section 4 quantifies the model using sector-level data for the G7 countries over the period 1978-2007. Implementing the decomposition of the overall comovement into the correlated shocks and transmission components from Section 2 requires the model solution (most importantly the influence matrix) and a time series for the vector of shocks. The model solution in turn requires two sets of objects: the input network (final and intermediate expenditure shares), and structural elasticities (of labor supply and substitution). The global input network comes from a standard source, the World Input-Output Database (WIOD). We use the model-implied relationships to structurally estimate the key elasticities. An important byproduct of this step is that it also yields confidence intervals for the values of these elasticities, that we use in the sensitivity analysis to provide a range of likely outcomes.

We then use the fully calibrated quantitative framework as a means to recover the shocks. As the global influence matrix translates sector-country specific shocks to equilibrium changes in output, it can also be inverted to infer supply shocks that rationalize the observed output changes. By construction, when these shocks are fed back into the model, they reproduce each country's GDP, and hence observed international GDP correlations exactly.[2]

[2]Foerster, Sarte, and Watson (2011) perform a related exercise in the closed economy. An advantage of our approach is that the model plus these shocks match the GDP data perfectly, which permits a decomposition of the observed GDP correlations into the different components. The disadvantage is that the resulting recovered shocks have a broad interpretation. In our framework, TFP and factor supply shocks have the same effect on the global vector of output changes, up to a scaling factor. Thus, inverting the global influence matrix yields a composite supply shock, which is sufficient to answer the main question

Our main finding is that the shocks required to rationalize observed output growth are correlated across countries. In our preferred calibration the transmission component accounts for about one-fifth of the total GDP correlation, with the shock correlation responsible for the remaining four-fifths. Thus, while network linkages do propagate shocks across borders, the internal transmission mechanisms in global production network models cannot generate all of the observed comovement. We also decompose overall GDP correlation into the components due to the contemporaneous change in GDPs due to a vector of shock innovations, and the infinite sum of responses to all the past innovations. Quantitatively, the intertemporal propagation through capital accumulation is much less important for comovement than the contemporaneous responses to shocks in the production network.

We next investigate the roles of the structural elasticities and the input network in generating this baseline result. The model requires 3 elasticities: the Frisch elasticity of labor supply, and substitution elasticities between inputs and final goods. The substitution elasticities govern both the direction and the magnitude of the demand shift experienced by the home economy following a foreign productivity shock. A higher Frisch elasticity implies a larger response of labor inputs to a given shift in demand, and hence a larger real GDP response. Lower substitution elasticities and higher Frisch elasticities thus lead to a greater share of transmission in total comovement.

We use our econometric estimates to obtain a three-dimensional empirical distribution over the three elasticities. We re-do the shock recovery procedure and calculate the share of overall comovement due to transmission and correlated shocks for each candidate set of elasticities. The result is an estimation-based distribution of the shares of transmission and correlated shocks in total comovement. By construction, the procedure matches the data on the overall GDP correlations under each set of elasticities, but the fraction of comovement attributed to transmission varies with these parameters. Most of the mass is in the range where shock correlation accounts for over 70% of the total comovement. Thus, our main result that shock transmission cannot generate nearly the observed level of comovement is robust to reasonable statistical parameter uncertainty.

Next, we investigate the role of international trade by comparing the baseline economy to one in which countries are in autarky. We write the difference in GDP comovement between the trade and autarky equilibria as a sum of two terms: the international transmission of shocks; and the change in contribution of correlated domestic shocks. The second term is the aggregation of the changes in the influence of the domestic shocks times the covariances of those shocks. As expected, international transmission is positive in the trade equilibrium and increases comovement relative to autarky. On the other hand, the second term tends to be negative and quantitatively important, reducing G7 GDP correlations by 0.05-0.1 on average. This effect has not to our knowledge been previously pointed out.

posed in Section 4. As discussed in detail in Section 5.2, many "demand" shocks such as monetary policy or sentiments manifest themselves in reduced form as factor supply shifts, and would thus be picked up in our composite sectoral supply shocks. Section 5.2 separates out several distinct types of shocks. Doing so leaves the main conclusion about the relative importance of correlated shocks vs. transmission in overall comovement virtually unchanged. An alternative use of our framework would be to feed in externally specified shocks for any subset of countries and sectors, which is appropriate in some applications (see, for example Bonadio et al., 2021a).

It occurs because opening to trade makes economies less susceptible to domestic shocks, reducing their influence on domestic GDP growth. Intuitively, when a country sources a significant share of expenditure abroad, domestic conditions matter less and domestic shocks propagate less to GDP. As a result, it matters less that domestic shocks are positively correlated across countries. Of course, this is a *ceteris paribus* effect. Domestic influence is replaced by foreign influence when countries open to trade, and comovement increases overall. But it increases by less than naive reasoning would imply.

Section 5 presents several extensions. First, we perform the analysis in a static model that is an international extension of Acemoglu et al. (2012). One advantage of the static approach is lower data requirements. Since to implement the static model we do not need to take a stand on forward-looking decisions, we do not need to estimate the joint stochastic process for the shocks in all countries and sectors. Thus, the static model can be implemented even on relatively short time series, and we can expand the analysis to include 29 countries available in KLEMS from 1995. The conclusion of the static analysis regarding the share of transmission in overall comovement is similar to the dynamic model implemented on the G7 countries.

The baseline analysis is based on a parsimonious framework with one composite supply shock. By construction, is not especially informative on the underlying drivers of business cycles in general, and of international comovement in particular. In the second extension we enrich the model to feature 4 shocks: TFP, labor, capital/investment, and intermediate input. We recover these 4 shocks to match the data on output, labor, capital, and intermediate inputs. The relative importance of correlated shocks vs. transmission is unchanged when we move to the 4-shock model. We simulate the model conditional on subsets of shocks to understand which ones are most important for comovement. No single shock has a dominant role in international comovement. Individually, the labor and the TFP shocks appear most promising.[3] A model that combines labor and TFP shocks strikes a good balance between parsimony and fit to the data. The two shocks together generate 80% of the observed international correlation, and produce behavior of GDP quite similar to the data. This specification is parsimonious both in the sense that it relies on only two shocks, as well as in the sense that these shocks themselves are relatively simple, and would work in the same way in a variety of models.

Finally, we develop further extensions incorporating wage rigidities and international financial integration. We provide an analytical characterization of how the parameter governing the extent of wage rigidity affects the impact response of the world economy to a vector of shocks. In our framework, greater wage rigidity acts like a higher Frisch elasticity, amplifying the economy's response to a given size shock, and thus the importance of transmission. However, even for quite high levels of wage rigidity transmission accounts for a minority of the overall comovement. We next implement a complete markets version of the model. If anything, introducing international risk

[3]Our labor supply shock can be viewed as a generalization of the "labor wedge" (e.g. Chari, Kehoe, and McGrattan, 2007) to the sector level. Though reduced-form, it has a variety of "demand shock" microfoundations, such as sentiment shocks (e.g. Angeletos and La'O, 2013; Huo and Takayama, 2015), monetary policy shocks under sticky wages (Galí, Gertler, and López-Salido, 2007; Chari, Kehoe, and McGrattan, 2007), or shocks to working capital constraints (e.g. Neumeyer and Perri, 2005; Mendoza, 2010).

sharing weakens international transmission of shocks to GDP, as it further decouples consumption from labor supply (and hence output). All in all, while these extensions do not overturn the main quantitative conclusions, introducing wage rigidities does increase the importance of transmission in international comovement, and thus should be explored in depth in future research.

Related Literature. Our paper draws from, and contributes to two literatures. The first is the active recent research agenda on shock propagation in production networks. A number of closed-economy papers following the seminal contributions of Carvalho (2010) and Acemoglu et al. (2012) enrich the theory, provide econometric evidence, and estimate key structural parameters (see, among others, Foerster, Sarte, and Watson, 2011; Acemoglu, Akcigit, and Kerr, 2016; Barrot and Sauvagnat, 2016; Atalay, 2017; Grassi, 2017; Baqaee, 2018; Baqaee and Farhi, 2019a,b; Boehm, Flaaen, and Pandalai-Nayar, 2019; Adao, Arkolakis, and Esposito, 2020; Allen, Arkolakis, and Takahashi, 2020; Bigio and La'O, 2020; Carvalho et al., 2020; Ferrari, 2022; Foerster et al., 2022; vom Lehn and Winberry, 2022). We apply the insights and tools developed by this body of work to the study of international GDP comovement. The notion that international input trade is a key feature of the global economy goes back to Hummels, Ishii, and Yi (2001) and Yi (2003), and has more recently been documented and quantified in a series of contributions by Johnson and Noguera (2012, 2017) and Caliendo and Parro (2015).[4]

The second is the research program in international macro that studies business cycle comovement using dynamic IRBC models featuring simple production structures. A large literature builds models in which fluctuations are driven by productivity shocks, and asks under what conditions those models can generate observed international comovement (see, among many others, Backus, Kehoe, and Kydland, 1992; Heathcote and Perri, 2002). A smaller set of contributions adds non-technology shocks (Stockman and Tesar, 1995; Wen, 2007; Bai and Ríos-Rull, 2015).[5] More recently, Ho, Sarte, and Schwartzman (2022) study the propagation of various idiosyncratic country shocks in a global New Keynesian model with both trade linkages and nominal rigidities, and extend the analysis to inflation comovement.

Our framework nests the rich static production network models, the canonical IRBC models, as well as more recent frameworks (such as Burstein, Kurz, and Tesar, 2008; Johnson, 2014; Eaton et al., 2016; Eaton, Kortum, and Neiman, 2016) that combine dynamics with simplified input-output structures. Note that the accounting decompositions of transmission vs. shock correlation and the

[4]In the international trade literature, contemporaneous work by Baqaee and Farhi (2019c) and subsequent work by Kleinman, Liu, and Redding (2020, 2022) also derives analytical first-order solutions to international network models. While these frameworks focus on long-run comparative statics such as gains from trade or foreign productivity growth, they cannot be used to study international transmission of business cycle shocks (and related applications). Because these papers feature fixed factor supply, measured real GDP is not responsive to foreign shocks, and thus international transmission (to real GDP) is nonexistent by construction.

[5]A number of papers are dedicated to documenting international correlations in productivity shocks and factor inputs (e.g. Imbs, 1999; Kose, Otrok, and Whiteman, 2003; Ambler, Cardia, and Zimmermann, 2004). Also related is the body of work that identifies technology and demand shocks in a VAR setting and examines their international propagation (e.g. Canova, 2005; Corsetti, Dedola, and Leduc, 2014; Levchenko and Pandalai-Nayar, 2020).

static vs. dynamic components developed in Section 2 also apply to all of these. While all papers on international business cycle comovement must take a stand on the relative importance of correlated shocks vs. transmission, we provide a method to cleanly separate these two potential sources of comovement that can be applied across models. We further contribute to this research agenda by deriving a set of analytical results that help quantify the relative importance of transmission and correlated shocks, measuring the shocks, and expanding the scope of quantification to more countries and sectors.[6,7]

2. Accounting

Consider an economy comprised of J sectors indexed by j and i, and N countries indexed by n and m. Gross output in sector j of country n at time t aggregates a primary factor input bundle $\mathcal{I}_{nj,t} \in \mathbb{R}_+$ (for instance, capital and labor) and materials inputs $X_{nj,t} \in \mathbb{R}_+$:

$$Y_{nj,t} = F_{nj}\left(\mathcal{I}_{nj,t}(\boldsymbol{\theta}), X_{nj,t}(\boldsymbol{\theta}); \boldsymbol{\theta}\right).$$

The bundle of inputs $X_{nj,t}$ can include foreign imported intermediates. The sectoral output is affected by a generic array of current and past shocks $\boldsymbol{\theta}$.[8] For concreteness, one can think of productivity shocks. A productivity shock $\theta_{nj,t}$ to sector j in country n at time t will directly affect output in that sector. Because the economy is interconnected through trade, output in every sector and country is in principle a function of all the history of shocks anywhere in the world, hence the dependence of $Y_{nj,t}$ on the full array $\boldsymbol{\theta}$ across countries, sectors, and time up to t. The array $\boldsymbol{\theta}$ can include multiple types of shocks (such as technology and non-technology). The next section completely specifies the shocks, and the nature of output's dependence on those shocks in the context of a particular model.

Following national accounting conventions, real GDP of country n at time t is defined as value added evaluated at the prices of some base year b:

$$G_{n,t} = \sum_{j=1}^{J}\left(P_{nj,b}Y_{nj,t}(\boldsymbol{\theta}) - P^X_{nj,b}X_{nj,t}(\boldsymbol{\theta})\right), \tag{2.1}$$

[6] Also related is the large empirical and quantitative literature on the positive association between international trade and comovement (e.g. Frankel and Rose, 1998; Imbs, 2004; Kose and Yi, 2006; di Giovanni and Levchenko, 2010; Ng, 2010; Liao and Santacreu, 2015; di Giovanni, Levchenko, and Mejean, 2018; de Soyres and Gaillard, 2019; Drozd, Kolbin, and Nosal, 2021). While these papers focus on the slope of the trade-correlation relationship in a cross-section of countries, we broaden the scope to provide a complete treatment of international comovement. Appendix D.5 explores the connection between the "trade-comovement" regressions and our analysis.

[7] Following the network propagation literature, our analysis captures the shock transmission through the market for inputs. It leaves open the possibility that the presence of input trade endogenously leads to correlated shocks, for instance through coordination of monetary policy, flow of information/sentiments, or transmission of productivity shocks within multinationals, among others. Microfounding shock correlation is outside the scope of our analysis but remains a fruitful avenue for future research.

[8] The shock $\boldsymbol{\theta} \in (\ell^2)^{NJ}$ as we allow $\boldsymbol{\theta}$ to have an infinite history for each country sector.

where $P_{nj,b}$ is the price of gross output and $P^X_{nj,b}$ is the price of the input bundle in the base year b.

Let $\theta_{mi,t}$ be a scalar-valued shock affecting sector i in country m at time t. A first order approximation to the log change in real GDP of country n can be written as:[9]

$$d \ln G_{n,t} = \sum_{k=0}^{\infty} \sum_{m} \sum_{i} s_{mni,k} \theta_{mi,t-k}, \tag{2.2}$$

where $s_{mni,k}$ are the elements of the global influence matrix, that give the elasticity of the GDP of country n with respect to shocks in sector i, country m, k periods in the past: $s_{mni,k} \equiv d \ln G_{n,t}/d\theta_{mi,t-k}$. Notice that these elasticities capture the full impact of a shock through direct and indirect input-output links and general equilibrium effects.

To highlight the sources of international GDP comovement, write real GDP growth as

$$d \ln G_{n,t} = \underbrace{\sum_{k=0}^{\infty} \sum_{j} s_{nnj,k} \theta_{nj,t-k}}_{\mathcal{D}_{n,t}} + \underbrace{\sum_{k=0}^{\infty} \sum_{j} s_{mnj,k} \theta_{mj,t-k}}_{\mathcal{P}_{n,t}} + \underbrace{\sum_{k=0}^{\infty} \sum_{n' \neq n,m} \sum_{j} s_{n'nj,k} \theta_{n'j,t-k}}_{\mathcal{T}_{n,t}}. \tag{2.3}$$

This equation simply breaks out the triple sum in (2.2) into the component due to country n's own shocks ($\mathcal{D}_{n,t}$), the component due to a particular trading partner m's shocks ($\mathcal{P}_{n,t}$), and the impact of "third" countries that are neither n nor m ($\mathcal{T}_{n,t}$).[10]

We assume that the world is stationary, and thus the moments of the GDP distributions do not depend on calendar time t. We interpret the $\theta_{nj,t}$'s as the innovations to the stochastic process for the exogenous states of the economy, and assume that these innovations have a covariance matrix $\mathbf{\Sigma}$ across country-sectors, but are uncorrelated across time.[11] Then, the GDP correlation between country n and country m is:

$$\begin{aligned} \varrho_{nm} \quad = \quad & \underbrace{\frac{\text{Cov}(\mathcal{D}_n, \mathcal{D}_m)}{\sigma_n \sigma_m}}_{\text{Shock Correlation}} + \underbrace{\frac{\text{Cov}(\mathcal{D}_n, \mathcal{P}_m) + \text{Cov}(\mathcal{P}_n, \mathcal{D}_m) + \text{Cov}(\mathcal{P}_n, \mathcal{P}_m)}{\sigma_n \sigma_m}}_{\text{Bilateral Transmission}} \\ + \quad & \underbrace{\frac{\text{Cov}(\mathcal{D}_n + \mathcal{P}_n + \mathcal{T}_n, \mathcal{T}_m) + \text{Cov}(\mathcal{T}_n, \mathcal{D}_m + \mathcal{P}_m)}{\sigma_n \sigma_m}}_{\text{Multilateral Transmission}}, \end{aligned} \tag{2.4}$$

[9]The first-order approximation converges to the exact change for infinitesimally small shocks. To streamline notation we use the equal sign instead of "≈" throughout the paper even when describing first-order model solutions. The extension to vector-valued $\theta_{mi,t}$ is straightforward, i.e. each sector can experience multiple shocks simultaneously.

[10]To be fully precise, $\mathcal{P}_{n,t}$ and $\mathcal{T}_{n,t}$ also depend on trading partner m. To avoid cluttering notation, we do not index these objects by m. This omission does not create an ambiguity in this section, as we only consider the correlation between n and m.

[11]That is, $\text{Cov}(\theta_{mi,t}, \theta_{nj,t'}) = 0 \; \forall t' \neq t$. This is without much loss of generality. Assuming that the shock innovations are uncorrelated across time still allows for a rich autocorrelation structure in the vector of exogenous states. For instance, if $\theta_{mi,t}$ is the innovation in the stochastic process for TFP growth, the global productivity vector can still follow an AR(p) process, with autoregressive coefficients on both the own sector's lagged TFP growth as well as spillover coefficients from other sectors' lagged TFP. The effects of persistent states are encoded in the elements of the influence matrix $s_{mnj,k}$.

where σ_n is the standard deviation of country n's GDP growth.

This expression underscores the sources of international comovement. The first term captures the fact that economies might be correlated even in the absence of trade if the underlying shocks themselves are correlated, especially in sectors influential in the two economies. The numerator of the Shock Correlation term can be written as:

$$\text{Cov}(\mathcal{D}_n, \mathcal{D}_m) = \sum_{k=0}^{\infty} \sum_j \sum_i s_{nnj,k} s_{mmi,k} \text{Cov}(\theta_{nj}, \theta_{mi}).$$

The second term captures bilateral or direct transmission. If the GDP of country n has an elasticity with respect to the shocks occurring in country m ($s_{mni,k} > 0$), that would contribute to comovement as well. Taking one of the components of the Bilateral Transmission component:

$$\begin{aligned} \text{Cov}(\mathcal{D}_n, \mathcal{P}_m) &= \sum_{k=0}^{\infty} \sum_j \sum_i s_{nnj,k} s_{nmi,k} \text{Cov}(\theta_{nj}, \theta_{ni}) \\ &= \sum_{k=0}^{\infty} \boldsymbol{s}'_{nn,k} \boldsymbol{\Sigma}_n \boldsymbol{s}_{nm,k}, \end{aligned}$$

where $\boldsymbol{\Sigma}_n$ is the $J \times J$ covariance matrix of shocks in country n, and $\boldsymbol{s}_{nm,k}$ is the $J \times 1$ influence vector collecting the impact of k-period lagged shocks in n on GDP in m. This expression underscores that one source of comovement is that under trade, both country n and country m will be affected by shocks in n.

Finally, the Multilateral Transmission term collects all the other sources of comovement between n and m that do not come from shocks to either n or m, such as shocks in other countries.

The overall comovement also admits an additive decomposition into the contributions of contemporaneous and past shocks. The GDP correlation between countries n and m can be written as:

$$\varrho_{nm} = \sum_{k=0}^{\infty} \frac{\boldsymbol{s}'_{n,k} \boldsymbol{\Sigma} \boldsymbol{s}_{m,k}}{\sigma_n \sigma_m}, \tag{2.5}$$

where $\boldsymbol{s}_{n,k}$ is the $NJ \times 1$ influence vector collecting the impact of all worldwide innovations k periods ago on country n. This leads to the following decomposition:

$$\varrho_{nm} = \sum_{k=0}^{\infty} \omega_{nm,k} \, \varrho_{nm,k}, \tag{2.6}$$

where

$$\varrho_{nm,k} = \frac{\boldsymbol{s}_{n,k} \boldsymbol{\Sigma} \boldsymbol{s}'_{m,k}}{\sqrt{\boldsymbol{s}_{n,k} \boldsymbol{\Sigma} \boldsymbol{s}'_{n,k}} \sqrt{\boldsymbol{s}_{m,k} \boldsymbol{\Sigma} \boldsymbol{s}'_{m,k}}} \qquad \text{and} \qquad \omega_{nm,k} = \frac{\sqrt{\boldsymbol{s}_{n,k} \boldsymbol{\Sigma} \boldsymbol{s}'_{n,k}} \sqrt{\boldsymbol{s}_{m,k} \boldsymbol{\Sigma} \boldsymbol{s}'_{m,k}}}{\sqrt{\sum_{k'=0}^{\infty} \boldsymbol{s}_{n,k'} \boldsymbol{\Sigma} \boldsymbol{s}'_{n,k'}} \sqrt{\sum_{k'=0}^{\infty} \boldsymbol{s}_{m,k'} \boldsymbol{\Sigma} \boldsymbol{s}'_{m,k'}}}.$$

In words, $\varrho_{nm,k}$ is the GDP correlation that would obtain due exclusively to shocks k periods ago. The weight $\omega_{nm,k}$ is the standard deviation of GDP only due to shocks k periods ago divided by the actual standard deviation of GDP (due to shocks at all lags).

Thus, the overall GDP correlation is additive in the component due to the contemporaneous shock innovations $\omega_{nm,0}\varrho_{nm,0}$ and the dynamic propagation of past shocks. In a static model, or more broadly any model without delayed propagation of shocks, the actual GDP correlation coincides with the contemporaneous component: $\varrho_{nm} = \varrho_{nm,0}$. Given a quantifiable model of delayed shock propagation, (2.6) gives a transparent answer to the question of how much comovement occurs due to contemporaneous vs. delayed effects of the shocks. We will quantify this decomposition below.

To summarize, in order to provide an account of international comovement, we must (i) recover shocks in order to understand their comovement properties; and (ii) assess how the global production and trade network (the distribution of $s_{nnj,k}$'s) translates sectoral comovement of the primitive shocks into GDP comovement. Finally, (iii) we must discipline the persistence of both the shocks and equilibrium adjustments over time in order to quantify the relative importance of contemporaneous vs. intertemporal correlation.

3. Theory

The decompositions above are general and would apply in any production economy. However, any measurement of the elements of the influence matrix and of shocks requires additional theoretical structure. This section introduces a parsimonious dynamic multi-country production network model and derives a number of analytical results. Section 4 uses it to quantify the contributions of correlated shocks and transmission to GDP comovement.

3.1 Setup

Households. Each country n is populated by an infinitely-lived representative household. The household consumes the final good available in country n and supplies labor and capital to firms. It solves

$$\max_{C_{n,t},\{I_{nj,t}\},\{H_{nj,t}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \, U\left(C_{n,t} - \sum_j H_{nj,t}^{1+\frac{1}{\psi}}\right) \tag{3.1}$$

subject to

$$P_{n,t}\left(C_{n,t} + \sum_j I_{nj,t}\right) = \sum_j W_{nj,t} H_{nj,t} + \sum_j R_{nj,t} K_{nj,t} \tag{3.2}$$

$$K_{nj,t+1} = (1-\delta_j) K_{nj,t} + I_{nj,t} \qquad \forall j, \tag{3.3}$$

where $C_{n,t}$ is consumption of final goods, $H_{nj,t}$ is the total labor hours supplied to sector j, and $K_{nj,t}$ is the amount of installed capital in sector j. Labor collects a sector-specific wage $W_{nj,t}$, and capital is rented at the price $R_{nj,t}$. As is customary, it takes one period for investment to become capital, and thus the capital stock in each sector is predetermined by one period.

Our formulation of the disutility of the labor supply is based on the Greenwood, Hercowitz, and Huffman (1988) preferences. The GHH preferences mute the interest rate effects and income effects on the labor supply, which helps to study the properties of the intratemporal equilibrium where the amount of capital is treated as predetermined. Labor and capital are differentiated by sector, as the household supplies factors to each sector separately. In this formulation, labor is neither fixed to each sector nor fully flexible, and its responsiveness is determined by the Frisch elasticity ψ.[12]

Our benchmark model assumes financial autarky, for two main reasons. First, as highlighted by Heathcote and Perri (2002), among others, models with financial autarky perform well in accounting for business cycle comovement.[13] Second, under financial autarky there is an analytical solution for the contemporaneous response of output to shocks (Proposition 3.1), that requires only observed export and import shares, the elasticities of substitution among intermediate and final goods, and the Frisch elasticity. Adding endogenous capital flows would come at a cost of both tractability and transparency. Section D.6 presents an extension with complete markets.

The final use in the economy, denoted $\mathcal{F}_{n,t} = C_{n,t} + \sum_j I_{nj,t}$, is an Armington aggregate across countries and sectors. Trade is subject to iceberg costs τ_{mnj} to ship good j from country m to country n (throughout, we adopt the convention that the first subscript denotes source, and the second destination). The functional form and its associated price index are given by

$$\mathcal{F}_{n,t} = \prod_j \left[\sum_m \vartheta_{mnj}^{\frac{1}{\rho}} \mathcal{F}_{mnj,t}^{\frac{\rho-1}{\rho}} \right]^{\nu_{nj} \frac{\rho}{\rho-1}}, \qquad P_{n,t} = \prod_j \left[\sum_m \vartheta_{mnj} \left(\frac{P_{mnj,t}}{\nu_{nj}} \right)^{1-\rho} \right]^{\frac{\nu_{nj}}{1-\rho}}, \tag{3.4}$$

where $\mathcal{F}_{mnj,t}$ is final use in n of sector j goods coming from country m, and $P_{mnj,t}$ is the price of $\mathcal{F}_{mnj,t}$. That is, the final bundle is Cobb-Douglas over sectoral bundles, and sectoral bundles are Armington aggregates across source countries. The expenditure shares on final good j imported form country m

[12]The specification of labor supply bears an affinity to the "Roy-Frechet" models common in international trade (e.g. Galle, Rodríguez-Clare, and Yi, 2022), in the sense that the relative supply of hours to two different sectors is isoelastic in the relative wages in the two sectors. The difference is that in most existing Roy-Frechet implementations, aggregate labor supply is fixed and only sectoral shares vary, whereas in our analysis total economywide labor supply shifts as well.

[13]We can easily accommodate a sequence of exogenous trade imbalances as in Dekle, Eaton, and Kortum (2008), without much change in the results. The financial autarky assumption is also adopted in Corsetti, Dedola, and Leduc (2008), Ruhl (2008), and many others. Kose and Yi (2006) show that when it comes to accounting for the trade-comovement relationship, the benchmarks of complete markets and financial autarky deliver similar results. We acknowledge that the financial autarky assumption excludes transmission mechanisms that operate through international capital flows. While this paper focuses on shock transmission through goods trade and production linkages, we leave the evaluation of other transmission mechanisms for future research.

within sector j and across all goods are given by

$$\pi^c_{mnj,t} = \frac{\vartheta_{mnj} P^{1-\rho}_{mnj,t}}{\sum_k \vartheta_{knj} P^{1-\rho}_{knj,t}}, \quad \text{and} \quad \pi^f_{mnj,t} = \nu_{nj} \pi^c_{mnj,t}. \tag{3.5}$$

The labor supply curves are isoelastic in the wages relative to the consumption price index, and given by (up to a normalization constant):

$$H^{\frac{1}{\psi}}_{nj,t} = \frac{W_{nj,t}}{P_{n,t}}. \tag{3.6}$$

The Euler equations governing investment in each sector j are:

$$U'_{n,t} = \beta \mathbb{E}_t \left[\left(\frac{R_{nj,t+1}}{P_{n,t+1}} + (1-\delta) \right) U'_{n,t+1} \right].$$

Firms. Sector j in country n is populated by competitive firms that operate a CRS production function

$$Y_{nj,t} = Z_{nj,t} \left(K^{1-\alpha_j}_{nj,t} H^{\alpha_j}_{nj,t} \right)^{\eta_j} X^{1-\eta_j}_{nj,t}, \tag{3.7}$$

where $Z_{nj,t}$ is the total factor productivity, and the intermediate input usage $X_{nj,t}$ is an aggregate of inputs from potentially all countries and sectors:

$$X_{nj,t} \equiv \left(\sum_i \sum_m \mu^{\frac{1}{\varepsilon}}_{mi,nj} X^{\frac{\varepsilon-1}{\varepsilon}}_{mi,nj,t} \right)^{\frac{\varepsilon}{\varepsilon-1}},$$

where $X_{mi,nj,t}$ is the usage of inputs coming from sector i in country m in production of sector j in country n, and $\mu_{mi,nj}$ is a taste shifter.[14]

Let $P_{mi,t}$ denote the price of output produced by sector i in country m, and let $P_{mi,nj,t}$ be the price paid in country-sector (n,j) for inputs from (m,i). No arbitrage in shipping implies that the price "at the factory gate" and the price at the time of final or intermediate usage are related by:

$$P_{mi,nj,t} = P_{mni,t} = \tau_{mni} P_{mi,t}.$$

[14] Liao and Santacreu (2015) show that in the presence of profits, international input trade will synchronize TFPs across countries, as measured by the Solow residual. A model with profits affecting the measured Solow residual is observationally equivalent to a model with decreasing returns to scale (see, e.g. Ruzic and Ho, 2021, for a discussion). In related work (Huo, Levchenko, and Pandalai-Nayar, 2020b) we extend our framework to estimate returns to scale and find them to be quite close to constant. Thus, in this paper we abstract from the role of profits in GDP synchronization.

Cost minimization implies that the payments to primary factors and intermediate inputs are:

$$\begin{aligned} R_{nj,t}K_{nj,t} &= (1-\alpha_j)\eta_j P_{nj,t}Y_{nj,t} \\ W_{nj,t}H_{nj,t} &= \alpha_j \eta_j P_{nj,t}Y_{nj,t} \qquad (3.8) \\ P_{mi,nj,t}X_{mi,nj,t} &= \pi^x_{mi,nj}\left(1-\eta_j\right)P_{nj,t}Y_{nj,t}, \qquad (3.9) \end{aligned}$$

where $\pi^x_{mi,nj,t}$ is the share of intermediates from country m sector i in total intermediate spending by n, j, given by:

$$\pi^x_{mi,nj,t} = \frac{\mu_{mi,nj}\left(\tau_{mni}P_{mi,t}\right)^{1-\varepsilon}}{\sum_{k,\ell}\mu_{k\ell,nj}\left(\tau_{knl}P_{k\ell,t}\right)^{1-\varepsilon}}. \qquad (3.10)$$

Equilibrium. An equilibrium in this economy is a set of sequences of of goods and factor prices $\left\{P_{nj,t}, W_{nj,t}, R_{nj,t}\right\}_{t=0}^{\infty}$, factor allocations $\left\{K_{nj,t}, H_{nj,t}\right\}_{t=0}^{\infty}$, and goods allocations $\left\{Y_{nj,t}\right\}_{t=0}^{\infty}$, $\left\{\mathcal{F}_{mnj,t}, X_{mi,nj,t}\right\}_{t=0}^{\infty}$ for all countries and sectors such that (i) households maximize utility; (ii) firms maximize profits; and (iii) all markets clear.

The following market clearing condition has to hold for each country n sector j:

$$P_{nj,t}Y_{nj,t} = \sum_m P_{m,t}\mathcal{F}_{m,t}\pi^f_{nmj,t} + \sum_m\sum_i (1-\eta_i)P_{mi,t}Y_{mi,t}\pi^x_{nj,mi,t}. \qquad (3.11)$$

Meanwhile, trade balance implies that each country's final expenditure equals the sum of value added across domestic sectors

$$P_{m,t}\mathcal{F}_{m,t} = \sum_i \eta_i P_{mi,t}Y_{mi,t}. \qquad (3.12)$$

Note that once we know the share of value added in production η_j, the expenditure shares $\pi^f_{nmj,t}$ and $\pi^x_{nj,mi,t}$ for all n, m, i, j, we can compute the nominal output $P_{nj,t}Y_{nj,t}$ for all country-sectors (n, j) after choosing a numeraire good. There is no need to specify further details of the model, and we will utilize this property to derive the influence matrix.

3.2 Impact Response

Under the maintained assumptions, our model has the property that the contemporaneous response of output to a worldwide vector of productivity shocks can be obtained by solving the set of intratemporal optimality and market clearing conditions. Furthermore, we can solve for the contemporaneous response of output to a productivity shock analytically to first order. The following proposition summarizes this discussion and states the analytical solution. Denote by "ln" the log-deviation from steady state/pre-shock equilibrium. Let the vectors $\ln \mathbf{Z}_t$, $\ln \mathbf{K}_t$, and $\ln \mathbf{Y}_t$ of length NJ collect the worldwide sectoral productivity, capital, and output log changes.

Proposition 3.1. *The response of worldwide output* $\ln \mathbf{Y}_t$ *to the global vector of supply shocks* $\ln \mathbf{Z}_t$ *and changes* *in capital* $\ln \mathbf{K}_t$ *is to a first order approximation given by*

$$\ln \mathbf{Y}_t = \mathbf{\Lambda} \left(\ln \mathbf{Z}_t + \boldsymbol{\eta}(\mathbf{I} - \boldsymbol{\alpha}) \ln \mathbf{K}_t\right), \tag{3.13}$$

where

$$\mathbf{\Lambda} = \left(\mathbf{I} - \frac{\psi}{1+\psi}\boldsymbol{\alpha}\boldsymbol{\eta}\left(\mathbf{I} + \left(\mathbf{I} - \mathbf{\Pi}^f\right)\mathcal{P}\right) - (\mathbf{I} - \boldsymbol{\eta})\left(\mathbf{I} + (\mathbf{I} - \mathbf{\Pi}^x)\mathcal{P}\right)\right)^{-1}, \tag{3.14}$$

$\boldsymbol{\eta}$ *and* $\boldsymbol{\alpha}$ *are matrices of output elasticities,* $\mathbf{\Pi}^f$ *and* $\mathbf{\Pi}^x$ *are matrices of the pre-shock final consumption and intermediate shares, respectively, and* $\mathcal{P}$ *is a matrix that combines both structural elasticities and pre-shock spending shares.*[15]

Proof. See Appendix B. □

Since the capital is predetermined at time t, equations (3.13)-(3.14) illustrate that all we need to understand the contemporaneous response of worldwide output $\ln \mathbf{Y}_t$ to various sector-country productivity shocks in this quantitative framework are measures of steady state final goods consumption and production shares, as well as model elasticities. The matrix $\mathbf{\Lambda}$ is the impact influence matrix. It encodes the contemporaneous general equilibrium response of output in every sector-country to shocks in every sector-country, taking into account the full model structure and all direct and indirect links between the countries and sectors.

The influence matrix (3.13)-(3.14) resembles the typical solution of a network model, that writes the equilibrium change in output as a product of the Leontief inverse and the vector of shocks. Our expression also features a vector of shocks, and an inverse of a matrix that is more complicated due to the multi-country structure of our model combined with elastic factor supply and non-unitary elasticities of substitution.

The proof proceeds by manipulating the equilibrium conditions of the model. To highlight the nature of general equilibrium effects captured by the influence matrix, linearize the market clearing conditions (3.11) to obtain

$$\ln \mathbf{P}_t + \ln \mathbf{Y}_t = \underbrace{\left(\mathbf{\Psi}^f \mathbf{\Upsilon} + \mathbf{\Psi}^x\right)(\ln \mathbf{P}_t + \ln \mathbf{Y}_t)}_{\text{destination country output variation}} + \tag{3.15}$$
$$\underbrace{(1-\rho)\left(\operatorname{diag}\left(\mathbf{\Psi}^f \mathbf{1}\right) - \mathbf{\Psi}^f \mathbf{\Pi}^c\right)\ln \mathbf{P}_t}_{\text{consumption goods relative price variation}} + \underbrace{(1-\varepsilon)\left(\operatorname{diag}\left(\mathbf{\Psi}^x \mathbf{1}\right) - \mathbf{\Psi}^x \mathbf{\Pi}^x\right)\ln \mathbf{P}_t}_{\text{intermediate goods relative price variation}}$$

where the vector $\ln \mathbf{P}_t$ collects sector-level log-deviations in prices, $\mathbf{\Psi}^x$ and $\mathbf{\Psi}^f$ are matrices containing the steady-state export shares of intermediate and final goods, $\mathbf{\Pi}^c$ is a matrix of final consumption

[15] The $NJ \times NJ$ diagonal matrices $\boldsymbol{\eta}$ and $\boldsymbol{\alpha}$ collect the η_j's and α_j's. A typical element of $\mathbf{\Pi}^f$ is π^f_{mnj} and a typical element of $\mathbf{\Pi}^x$ is $\pi^x_{mi,nj}$. All of the matrices in Proposition 3.1 are defined precisely in Appendix B.

share for the inner layer of CES, and $\boldsymbol{\Upsilon}$ is a matrix of value added shares.[16] The first term contains the response of nominal output that arises from output changes in every country and sector following a shock. The second term contains the relative price changes of final goods and the final term the relative price changes of intermediate inputs. Equation (3.15) implies that we can solve for the vector of country-sector price changes as a function of output changes and a matrix $\mathcal{P}$ that depends only on spending shares and structural elasticities:

$$\ln \mathbf{P}_t = \mathcal{P} \ln \mathbf{Y}_t. \tag{3.16}$$

The matrix $\mathcal{P}$ is then an input into the influence matrix. Combining (3.16) with linearized versions of the production function (3.7), labor market clearing, and the demand for intermediate goods leads to the model solution (3.13).

GDP Change and Shock Transmission. Proposition 3.1 states the change in gross output, whereas GDP is value added. The following proposition describes the GDP changes.

Proposition 3.2. *The real GDP change in any country n is given by*

$$\ln G_{n,t} = \sum_{j=1} \omega_{nj} \left(\ln Z_{nj,t} + \alpha_j \eta_j \ln H_{nj,t} + (1-\alpha_j)\eta_j \ln K_{nj,t} \right), \tag{3.17}$$

where $\omega_{nj} \equiv \frac{P_{nj}Y_{nj}}{G_n}$ is the steady-state Domar weight, and the global vector of changes in hours is given by:

$$\ln \mathbf{H}_t \equiv \mathcal{H} \left(\ln \mathbf{Z}_t + (\boldsymbol{\eta} - \boldsymbol{\alpha\eta}) \ln \mathbf{K}_t \right), \quad \mathcal{H} = \frac{\psi}{1+\psi} \left(\mathbf{I} + \left(\mathbf{I} - \mathbf{\Pi}^f \right) \mathcal{P} \right) \boldsymbol{\Lambda}. \tag{3.18}$$

Proof. See Appendix B. □

To construct GDP, we need to aggregate the changes of sector-country real value added, as in (2.1). The first term in equation (3.17) captures the impact of domestic TFP changes on GDP. Note that there is no direct dependence of country n's GDP on foreign TFP changes. The second term in (3.17) captures the changes in hours. Equation (3.18) underscores that hours in every country and sector depend on the entire vector of TFP changes worldwide.

The international transmission mechanism in our model is thus the endogenous factor supply responses to foreign shocks. This is the main transmission mechanism in the international macro literature going back to its origins (e.g. Backus, Kehoe, and Kydland, 1992), that has been used to study shock transmission through production networks (e.g. Kose and Yi, 2006; Burstein, Kurz, and Tesar, 2008; Johnson, 2014). Our contribution here is to provide a transparent analytical solution to

[16] A typical element of $\boldsymbol{\Psi}^f$ is $\frac{P_m \mathcal{F}_m \nu_j \pi^f_{nmj}}{P_{nj} Y_{nj}}$, a typical element of $\boldsymbol{\Psi}^x$ is $\frac{P_{mi} Y_{mi} \pi^x_{nj,mi}}{P_{nj} Y_{nj}}$, a typical element of $\mathbf{\Pi}^c$ is π^c_{mnj}, and a typical element of $\boldsymbol{\Upsilon}$ is $\frac{\eta_i P_{mi} Y_{mi}}{P_m \mathcal{F}_m}$. See the proof of Proposition 3.1 in Appendix B for the detailed definitions.

a larger-scale international input network model, that enables a simple decomposition of the overall comovement into the transmission and shock correlation terms.[17]

Proposition 3.2 connects to the accounting decomposition (2.4) of comovement into shock correlation and transmission components. Mapping to the notation of Section 2, the contemporaneous GDP impact of a domestic TFP shock in sector j is equal to $s_{nnj,0} = \omega_{nj} + \sum_i \alpha_i \eta_i \omega_{ni} \mathcal{H}_{ni,nj}$, where $\mathcal{H}_{ni,nj}$ is the (ni, nj)th element of the hours influence matrix. The contemporaneous GDP impact of a shock to sector j in a foreign country m is given by $s_{mnj,0} = \sum_i \alpha_i \eta_i \omega_{ni} \mathcal{H}_{ni,mj}$. Thus, the direct effect of own TFP changes (the first term in 3.17) plus the block-diagonal elements of $\mathcal{H}$ together make up $\mathcal{D}_n$ – the influence of domestic shocks on GDP, and thus determine the shock correlation term. The off-diagonal elements of $\mathcal{H}$ capture the influence of foreign shocks on a country's GDP, and thus make up the transmission terms.

Intuition. To better understand the forces shaping the influence matrix, consider the following special case.

Corollary 3.3. *With unitary elasticities of substitution, the influence matrix for the impact response of hours is*

$$\begin{aligned} \mathcal{H} &= \frac{\psi}{1+\psi}\mathbf{\Pi}^f \left(\mathbf{I} - \frac{\psi}{1+\psi}\boldsymbol{\eta}\boldsymbol{\alpha}\mathbf{\Pi}^f - (\mathbf{I}-\boldsymbol{\eta})\mathbf{\Pi}^x\right)^{-1} \\ &= \frac{\psi}{1+\psi}\mathbf{\Pi}^f \sum_{k=0}^{\infty}\left(\frac{\psi}{1+\psi}\boldsymbol{\eta}\boldsymbol{\alpha}\mathbf{\Pi}^f + (\mathbf{I}-\boldsymbol{\eta})\mathbf{\Pi}^x\right)^{k}, \end{aligned} \tag{3.19}$$

and thus all the elements of $\mathcal{H}$ are non-negative.

Proof. See Appendix B. □

According to (3.19), the response of hours $\mathcal{H}$ can be built directly using the observed expenditure shares. Importantly, since all the elements in the final goods and intermediate input expenditure shares are non-negative, it follows that all the elements in $\mathcal{H}$ are positive. Combined with Proposition 3.2, this implies that under unitary elasticities the transmission terms are positive: a country's GDP increases following a positive foreign shock. Relatedly, if countries use foreign goods more intensively ($\mathbf{\Pi}^f$ or $\mathbf{\Pi}^x$ contain larger off-diagonal elements), the off-diagonal elements of $\mathcal{H}$ become larger and the international transmission channels more potent. Finally, this special case makes it transparent that a higher Frisch elasticity amplifies the off-diagonal elements in $\mathcal{H}$, resulting in greater international transmission.

[17] Proposition 3.2 highlights the difference between our analytical results and the contemporaneous work by Baqaee and Farhi (2019c) that considers the case of exogenous factor supplies. Proposition 3.2 under the assumption that $\ln H_{nj,t}$ and $\ln K_{nj,t}$ are exogenous is essentially Theorem 1 in that paper (and the earlier result in Kehoe and Ruhl, 2008). By contrast, we obtain an analytical solution when factor supplies are endogenous to shocks, and show that in this case a country's real GDP does respond to foreign shocks. Obviously, this property is essential to make the analysis of international shock transmission non-trivial. When factor supply is exogenous as in Baqaee and Farhi (2019c), international shock transmission to real GDP is ruled out by construction, and the entirety of international GDP comovement is trivially accounted for by shock correlation.

The second line of equation (3.19) writes the hours response as an infinite summation of the share matrices, reflecting first-, second-, etc. round effects propagating via relative price changes through the input and factor markets. This infinite propagation reflects the general-equilibrium (GE) forces, through which each sector's hours are generically a function of the entire global vector of productivity shocks. The term $\frac{\psi}{1+\psi}\boldsymbol{\eta}\boldsymbol{\alpha}\boldsymbol{\Pi}^f$ captures the GE multiplier due to the endogenous labor response, and the term $(\mathbf{I}-\boldsymbol{\eta})\boldsymbol{\Pi}^x$ captures the GE multiplier due to the adjustments of material inputs.[18]

Static Network Models. An active literature, initiated by Carvalho (2010) and Acemoglu et al. (2012), explores shock propagation in static production network models. It turns out that the contemporaneous output and GDP response to shocks in our fully dynamic model coincides with the response of a static network economy to the same shock.[19] This claim follows immediately from the proof of Proposition 3.1. The influence matrix $\boldsymbol{\Lambda}$ is obtained solely from the intratemporal optimality and market clearing conditions (3.6), (3.8)-(3.9), and (3.11)-(3.12), and thus requires no forward-looking equations or expectations. Thus, to understand today's response of output to today's shock, we do not need to take a stand on the future evolution of the economy following this shock, or evaluate agents' expectations over the future variables. The later periods' GDP response to today's shock will depend on the properties of the stochastic process for shocks and the capital accumulation decisions, which are not encoded in $\boldsymbol{\Lambda}$ (but can be evaluated numerically).

Our analysis thus integrates the static network propagation literature that follows Acemoglu et al. (2012) and the dynamic international business cycle literature. We can cleanly separate the instantaneous propagation analyzed in the former and the delayed responses to shocks emphasized by the latter. We will use this property in the analysis below. First, implementing the dynamic model requires observing a long time series of shocks, as we need to estimate the shock process in order to give it to the agents to form expectations. On the other hand, the static model does not require us to estimate the shock process, and so it can be implemented on any length time series. We only have long time series for a small number of countries, but data for 1995-2007 are available for 29 countries. Thus, in addition to reporting the dynamic model results for a small set of countries, we will also report the results under the static network model for a large sample of countries. For the subset of countries for which both the dynamic and static models can be implemented, they deliver very similar results, lending credibility to the static network model.

Second, we can compute the exact nonlinear static model solution using the Dekle, Eaton, and Kortum (2008) exact hat algebra, and evaluate the quality of the first-order approximation deployed

[18]The general versions of these terms are $\frac{\psi}{1+\psi}\boldsymbol{\alpha}\boldsymbol{\eta}\left(\mathbf{I}+\left(\mathbf{I}-\boldsymbol{\Pi}^f\right)\mathcal{P}\right)$ and $(\mathbf{I}-\boldsymbol{\eta})\left(\mathbf{I}+\left(\mathbf{I}-\boldsymbol{\Pi}^x\right)\mathcal{P}\right)$, respectively.

[19]This statement is of course holding fixed the vector of capital stocks and all the expenditure shares that enter the influence matrix. The static network model is characterized by the equations in Section 3.1, with (3.1) replaced by static optimization $\max_{C_{n,t},\{H_{nj,t}\}} U\left(C_{n,t}-\sum_j H_{nj,t}^{1+\frac{1}{\psi}}\right)$, setting $I_{nj,t}=0$ in (3.2) with $K_{nj,t}$ exogenously given, and dropping the capital accumulation equation (3.3). While the impact response of GDP and sectoral output coincide in the two models, trivially the responses of consumption and investment do not, as in the dynamic model the agents split final output into consumption and investment, whereas in the static model all final output is consumed.

throughout the paper. Appendix B.3 presents the details of the exact solution, and compares the GDP growth rates implied by the two approaches. The exact and first-order approximation solutions are very close to each other, as shown by Figure A3.[20]

3.3 Dynamics

With endogenous capital accumulation, the current output in a location not only depends on current global shocks but also the past global shocks. The following proposition states the general form of this type of dependence.

Proposition 3.4. *When the vector of supply shocks follows a first-order auto-regressive process, the equilibrium law of motion of capital obeys*

$$\ln \mathbf{K}_{t+1} = (\mathbf{I} - \mathbf{M}\,\mathbb{L})^{-1}\mathbf{\Gamma} \ln \mathbf{Z}_t,$$

where $\mathbb{L}$ *is the lag operator, and the matrices* $\mathbf{M}$ *and* $\mathbf{\Gamma}$ *depend on steady-state shares, elasticities of substitution, the discount factor, and the depreciation rates, and* $\mathbf{\Gamma}$ *also depends on the shock processes.*

Proof. See Appendix B. □

Propositions 3.1 and 3.4 imply that the entire history of global shocks matters for the current GDP and therefore international comovement. However, the impact of past shocks will fade away compared with the contemporaneous ones. In Section 4, we will quantify the contribution of this dynamic effect. In addition, the parsimonious structure discussed in Proposition 3.1 largely extends to the dynamic setting. There is no need to specify additional model details other than the depreciation rates, the discount factor, and the processes for the productivity shocks.

To further build intuition, we consider a special case where capital accumulation is subject to full depreciation ($\delta_j = 1\ \forall j$). In this case, the economy features constant saving rates with respect to the country's total income, which is a generalization of the textbook growth model result to an international economy with trade. The resulting dynamics of GDP changes can be summarized as follows.

Proposition 3.5. *When* $\delta_j = 1$, $\alpha_j = \alpha$, *and* $U(\cdot) = \ln(\cdot)$, *the GDP changes are given by*

$$\ln G_{n,t} = \sum_j \omega_{nj} \left(\ln Z_{nj,t} + (1-\alpha_j)\eta_j \ln G_{n,t-1} + \alpha_j \eta_j \sum_{m,i} \mathcal{H}_{nj,mi} \big(\ln Z_{mi,t} + \eta_i (1-\alpha_i) \ln G_{m,t-1} \big) \right).$$

Proof. See Appendix B. □

[20] Appendix B.3 also explores the difference between the exact and linearized models for varying shock sizes and shock correlations, and illustrates that in the quantitatively relevant range of shocks, the linear approximation is a very good fit.

The first two terms in the parentheses are the GDP responses to a country's own TFP shock and to their own past GDP changes. The term involving $\ln G_{n,t-1}$ simply captures the direct dependence of today's capital stock on last period's domestic capital decisions. The last term encodes the transmission of current and past shocks in other countries to country n's GDP. This system of N equations depicts the entire dynamics of global GDP changes.

With partial depreciation, the GDP dynamics are more involved and we explore their quantitative properties in the next section.

4. Quantification

This section quantifies the dynamic global network model laid out in the previous section to explore the nature of international GDP comovement.

4.1 Data and Calibration

To quantify the model, we require data on the (i) growth of real value added and hours worked for a panel of countries, sectors, and years and (ii) global input-output linkages. The dataset with the broadest coverage for real value added is KLEMS 2009 (O'Mahony and Timmer, 2009).[21] It contains gross output, value added, labor and capital inputs, as well as output and input deflators. The database covers all sectors of the economy at a level slightly more aggregated than the 2-digit ISIC revision 3, yielding, after harmonization, 30 sectors listed in Appendix Table A1. In a limited number of instances, we supplemented KLEMS with data from the WIOD Socioeconomic Accounts, which contains similar variables. The core of the analysis is carried out on the G7 countries for which we have a balanced panel over 1978-2007, and an aggregate of other countries into a "rest of the world." Sections 4.2 and 5.1 utilize a broader sample of 29 economies, for which data are available from the mid-1990s. Appendix Table A2 lists the countries.

Constructing the influence matrix (3.14) requires expenditure and sales shares $\mathbf{\Pi}^f$, $\mathbf{\Pi}^x$, $\mathbf{\Psi}^f$, and $\mathbf{\Psi}^x$, Cobb-Douglas shares α_j and η_j, and three elasticities: the two substitution elasticities ρ and ε, and the labor supply elasticity ψ. The data on input linkages at the country-sector-pair level, as well as on final goods trade come from the 2013 WIOD database (Timmer et al., 2015), which contains the global input-output matrix. The expenditure and sales shares $\mathbf{\Pi}^f$, $\mathbf{\Pi}^x$, and $\mathbf{\Psi}^f$ can be computed directly from WIOD. Capital shares in total output α_j and value added shares in gross output η_j come from KLEMS. We time-average the expenditure and sales share matrices, and average α_j and η_j in each sector across countries and time to reduce noise. The model period is a year. We set the discount rate to $\beta = 0.96$, and the depreciation rates δ_j are set to match sector-specific depreciation rates obtained from the BEA for 2001.[22] The period utility is $U(\cdot) = \ln(\cdot)$.

[21] This is not the latest vintage of KLEMS, as there is a version released in 2016. Unfortunately, the 2016 version has a shorter available time series, as the data start in 1995, and also has many fewer countries. A consistent concordance between the two vintages is not feasible without substantial aggregation.

[22] The BEA provides depreciation rates for 1995-2007 that can be mapped to NAICS codes. We concord these to the sectors

4.2 Elasticity Estimation

The elasticities ρ, ε, and ψ are estimated structurally by fitting model-implied relationships to data on expenditure shares, prices, and hours worked. This section summarizes the procedures briefly. Appendix C contains the detailed discussion.

To estimate ρ and ε, we log-difference the CES expenditure shares (3.5) and (3.10) with respect to $t-1$ and a reference country m'. This yields the following relationships between shares and prices:

$$\ln\left(\frac{\widehat{\pi}^{f}_{mnj,t}}{\widehat{\pi}^{f}_{m'nj,t}}\right) = (1-\rho)\ln\left(\frac{\widehat{P}_{mj,t}}{\widehat{P}_{m'j,t}}\right) + \ln\left(\frac{\widehat{\vartheta}_{mnj,t}\widehat{\tau}^{1-\rho}_{mnj,t}}{\widehat{\vartheta}_{m'nj,t}\widehat{\tau}^{1-\rho}_{m'nj,t}}\right)$$

and

$$\ln\left(\frac{\widehat{\pi}^{x}_{mj,ni,t}}{\widehat{\pi}^{x}_{m'j,ni,t}}\right) = (1-\varepsilon)\ln\left(\frac{\widehat{P}_{mj,t}}{\widehat{P}_{m'j,t}}\right) + \ln\left(\frac{\widehat{\mu}_{mj,ni,t}\widehat{\tau}^{1-\varepsilon}_{mnj,t}}{\widehat{\mu}_{m'j,ni,t}\widehat{\tau}^{1-\varepsilon}_{m'nj,t}}\right),$$

where the "hat" refers to the *gross* proportional change in any variable between time t and $t-1$. The structural error term is then the stochastic component of iceberg trade costs, final consumer taste shocks, and input share shocks, as well as any measurement error.

As the expenditure share changes $\widehat{\pi}$ are relative to a reference country m', the estimation amounts to regressing double-differenced expenditure share changes on relative price changes. A threat to identification would be that relative source country price changes are affected by destination country demand shocks (e.g. $\widehat{\vartheta}_{mnjt}$), and thus correlated with the residual. We address endogeneity in three ways. First, we include source-destination-reference country-time ($n \times m \times m' \times t$) fixed effects, which absorb many confounders including any common components occurring at the country 3-tuple-time level, such as exchange rate changes and other taste and transport cost changes. Thus the coefficient is estimated from the variation in the relative sectoral price indices and relative sectoral share movements within that cell. Second, our estimates are based on the subsample in which destination countries are all non-G7, and the source and reference countries are all G7 countries. Therefore, it is unlikely that taste shocks in the (smaller) destination countries will affect relative price changes in the larger G7 source countries. Third, we use foreign Solow residual shocks as instruments for changes in relative prices. The exclusion restriction is that the Solow technology shocks in source countries are uncorrelated with taste and trade cost shocks in the destination countries in the relevant sample and conditional on the fixed effects.[23]

Our preferred estimates are $\rho = 1.43$ and $\varepsilon = 0.89$. These values for annual frequency elasticities are sensible in light of existing estimates (Atalay, 2017; Boehm, Flaaen, and Pandalai-Nayar, 2019; Boehm, Levchenko, and Pandalai-Nayar, 2022). Appendix Table A4 contains the full battery of

in the WIOD. As the depreciation rates are relatively stable over time, we use the depreciation rates of 2001.

[23] Huo, Levchenko, and Pandalai-Nayar (2020a) find that the Solow residual is virtually uncorrelated across countries. Note that the Solow residual is not the same as the composite shock $\ln \mathbf{Z}_t$ in our model. As discussed above, the composite shock will include true TFP shocks as well as shocks to factor supply.

results, including extensive robustness checks, discussed in the text in Appendix C.1.

To estimate ψ, we use the responses of labor hours to shocks in a theory-consistent way. The starting point are the responses of hours to productivity shocks given by Proposition 3.2. However, because the Proposition states the general-equilibrium relationship, the vector of shocks $\ln \mathbf{Z}_t$ act as both labor demand shifters through their effect on $\ln \mathbf{Y}_t$, but also as labor supply and demand shifters through their effects on equilibrium final goods prices $\mathbf{\Pi}^f \ln \mathbf{P}_t$ and firm intermediate goods prices $\ln \mathbf{P}_t$. This is a threat to identification, as estimating the labor supply elasticity ψ requires isolating shocks that shift the labor demand curve but hold the labor supply curve approximately constant. An additional threat to identification is other shocks to labor supply. As a result, as is well known in the literature estimating the Frisch elasticity, we cannot simply estimate ψ by regressing hours on either observed or model-implied wages. In fact, if TFP shocks in some sectors increase using sector labor demand and also decrease the final goods price index for consumers, thereby decreasing labor supply, we would expect the estimate of ψ from regressing hours growth on wage changes to be biased towards zero.

To address this issue, we use a plausibly exogenous set of $\ln \mathbf{Z}_t$ shocks, that are likely to shift sectoral labor demand but not sectoral labor supply, and use them to estimate the labor supply elasticity. The procedure, detailed in Appendix C.2, is broadly based on Shea (1993a,b). Briefly, a shock to (m, i) is assumed to be a plausibly exogenous shifter for the labor demand in (n, j) if (m, i) is an important intermediate input ($\pi^x_{mi,nj}$ is large), but not an important final consumption good (π^f_{mnj} is small). The model-consistent estimation of ψ is conditional on the parameters ρ and ε estimated above. Further, because $\mathbf{\Lambda}$ is a function of ψ, the estimating equation is nonlinear, and so we utilize nonlinear least squares. Our baseline estimate of ψ is 0.72, close to the value of 0.75 recommended by Chetty et al. (2011) for use in macro models.

Table 1 summarizes the baseline parameters for the network model and data sources. Section 4.6 performs a systematic sensitivity analysis with respect to the values of ρ, ε and ψ. These are the key source of uncertainty in the quantification; the other model inputs, such as expenditure shares and production function parameters, are directly available in the data. Our estimation procedure yields the joint distribution of ρ, ε and ψ. In other words, we do not treat the uncertainty over these three parameters as independent. We instead estimate the joint likelihood of the different combinations of these three parameters.

4.3 Recovering Shocks

The core quantification exercise of the paper is a decomposition (2.4) of the overall GDP correlation into the shock correlation and transmission terms. The exercise is at its most informative when the model can replicate the observed GDP correlations. Thus, we invert the model to recover the global vector of supply shocks $Z_{nj,t}$ in such a way as to match actual value added growth in every country and sector (and therefore actual GDP growth in every country).

Table 1: Parameter Values

Param.	Value	Source	Related to
ρ	1.43	Appendix C.1	final substitution elasticity
ε	0.89	Appendix C.1	intermediate substitution elasticity
ψ	0.72	Appendix C.2	Frisch elasticity
β	0.96	standard	discount rate
δ_j	[0.07, 0.13]	BEA	depreciation rates
α_j	[0.40, 0.79]	KLEMS	labor and capital shares
η_j	[0.31, 0.67]	KLEMS	intermediate input shares
ν_{nj}	—	WIOD	final use sectoral expenditure shares
π^f_{mnj}	—	WIOD	final use trade shares
$\pi^x_{mi,nj}$	—	WIOD	intermediate use trade shares

Notes: This table summarizes the parameters and data targets used in the quantitative model, and their sources. For α_j, δ_j and η_j, the table reports the 10th and 90th percentiles of the range of these parameters.

Let the vector $\ln \mathbf{V}_t$ of length NJ collect sectoral value added in log deviations from steady state. Similar to Proposition 3.2, sectoral value added can also be expressed as a function of shocks and changes in primary inputs (derivation in Appendix B):

$$\begin{aligned}\ln \mathbf{V}_t &= \ln \mathbf{Z}_t + \boldsymbol{\alpha\eta} \ln \mathbf{H}_t + (\mathbf{I} - \boldsymbol{\alpha})\boldsymbol{\eta} \ln \mathbf{K}_t \\ &= (\mathbf{I} + \boldsymbol{\alpha\eta\mathcal{H}}) \ln \mathbf{Z}_t + (\mathbf{I} + \boldsymbol{\alpha\eta\mathcal{H}})(\mathbf{I} - \boldsymbol{\alpha})\boldsymbol{\eta}(\mathbf{I} - \mathbf{M}\mathbb{L})^{-1}\boldsymbol{\Gamma} \ln \mathbf{Z}_{t-1}. \end{aligned} \tag{4.1}$$

Thus, the structure of the model world economy and the observed/measured objects can be used to recover the history of the global vector of supply shocks $\{\ln \mathbf{Z}_{t-\tau}\}_{\tau=0}^{\infty}$ that rationalizes the observed real value added growth rates in each country-sector and year. Note that the interdependence between country-sectors through input linkages implies that the entire global vector $\{\ln \mathbf{Z}_{t-\tau}\}_{\tau=0}^{\infty}$ must be solved for jointly.

With fixed exogenous capital, as in a static network model, the procedure would be especially simple: (4.1) is inverted to get $\ln \mathbf{Z}_t = (\mathbf{I} + \boldsymbol{\alpha\eta\mathcal{H}})^{-1} \ln \mathbf{V}_t$.[24] The presence of endogenous capital accumulation makes this procedure more challenging as the matrices $\mathbf{M}$ and $\boldsymbol{\Gamma}$ depend on the shock processes. However, the shock processes can only be estimated when the shock realizations are available. Therefore, recovering the shocks requires finding a fixed point.

To proceed, we assume that the country-sector shocks follow a vector autoregressive process. Due to the large number of countries and sectors, it is not feasible to estimate a fully unrestricted VAR. We therefore impose parsimonious functional forms on the shock processes:

$$\ln Z_{nj,t} = \rho_{nj} \ln Z_{nj,t-1} + \zeta_n \mathbf{1}(m = n, k \neq j) \ln Z_{mk,t-1} + \theta_{nj,t}. \tag{4.2}$$

[24] We verify this matrix is invertible in our data.

These processes allow for own autocorrelation and within-country lagged spillovers of sectoral shocks. In addition, innovations $\theta_{nj,t}$ can have an arbitrary contemporaneous cross-border and cross-sector covariance structure. On the other hand, (4.2) does not allow for lagged cross-border spillovers.

Computationally, we adopt an iterative procedure. We start with a guess for the shock process, which allows us to compute the implied shock series. We then estimate the VAR processes (4.2), which leads to updated shock processes. We iterate these two steps to convergence. The procedure ensures that the perceived laws of motion are consistent with the actual ones.[25] The result is consistent with rational expectations, which imposes the cross-equation restriction that perceived shock processes need to coincide with actual shock processes.

At a formal level, the only shock in this world economy is the TFP shock $Z_{nj,t}$. From the perspective of this shock recovery procedure and the quantification that follows, $Z_{nj,t}$ should be interpreted broadly as a composite supply shock, encompassing both technology and primary factors. In the model, labor supply is upward-sloping in real wages, but shifts in the labor supply curve are isomorphic to TFP shocks in their effect on the global vector of output changes, up to a scaling factor. The dynamic model predicts a change in the next period's capital stock as a function of the history of shocks. If in the data, part of the reason for a high value added in the next period is a higher capital stock than what is implied by the model, the shock inversion will attribute this to a higher TFP. (Note that this procedure targets value added, not the capital stock series.) Thus, inverting the global influence matrix recovers a composite supply shock, which is sufficient to answer the main question posed in this section. Section 5.2 formally separates technology, labor supply, and capital supply shocks.

4.4 Impulse Responses

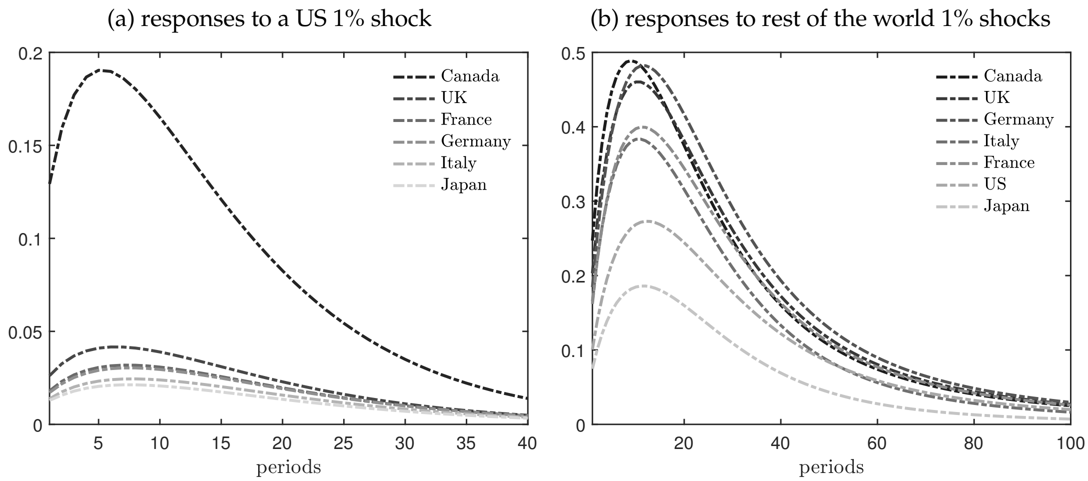
We start with a "test drive" of the propagation mechanism by computing the world economy's response to some simple hypothetical shocks (i) a 1% US shock in all sectors; and (ii) a 1% rest-of-the-world shock in all sectors from the perspective of each country. The rest-of-the-world exercise assumes that the country in question is not shocked, but all other possible countries and sectors are, and thus has to be conducted country by country.

The left panel of Figure 1 displays the change in real GDP in every other country in the world following a 1% US shock in each sector. The results show that the observed trade linkages do result in transmission. The smaller economy with the largest trade linkages to the US – Canada – is most strongly affected by the US shocks. The mean response of foreign GDP at the peak is 0.05%, and the maximum response – Canada – is about 0.16%.

Next, we simulate the real GDP responses of each country n in the sample when all other countries

[25] Further details of the estimation are in Appendix C.3. Appendix Table A5 summarizes the estimated shock processes. We have also experimented with shock processes that include lagged within-sector cross-border spillovers, and the quantitative results remain similar. We make the assumption that the shocks before the first available observation of value added are zero.

Figure 1: Impulse Responses to US and Rest-of-World Shocks



Notes: Panel (a) displays the change in log real GDP of every other country in the sample when the United States experiences a supply shock of 0.01 in every sector. Panel (b) displays the change in log real GDP of every country in the sample when the rest of the world excluding the country experiences a supply shock of 0.01 in every sector.

(excluding n) experience a 1% technology shock. This exercise answers the question, if there is a 1% world shock outside of the country, how much of that shock will manifest itself in the country's GDP? The right panel of Figure 1 displays the results. In response to a 1% outside world shock, the mean country's GDP increases by 0.34% at the peak, with the impact ranging from around 0.16% in Japan to around 0.43% in Canada. Not surprisingly, smaller and more open countries are more affected by shocks in their trade partners. All in all, these exercises suggest that outside world shocks have a significant impact on most countries.

4.5 Decomposing GDP Correlations

Table 2 implements the decomposition (2.4) of the overall GDP comovement into the shock correlation and transmission terms, using the shocks recovered in Section 4.3 to match the observed value added growth. The core sample of countries is the G7 over the period 1978-2007. These countries have sufficiently long time series data in KLEMS so that the dynamic model can be implemented. The first row reports the GDP growth correlations in the data, the second row in the model.[26] The rest of the table reports the results of the decomposition (2.4). The correlation of shocks is responsible for over four-fifths of the total comovement. Nonetheless, the bilateral and multilateral transmission terms

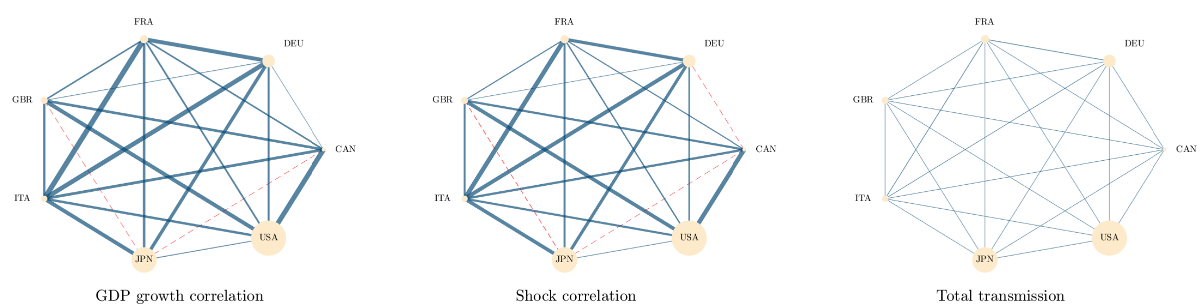
[26]Our procedure matches the sectoral value added growth rates in the data. The small discrepancy in GDP correlations between the data and the model is due to the fact that the model Domar weights are slightly different from the data. This is because in the model we must ensure trade balance, whereas in the data trade is unbalanced. The discrepancy introduced by this divergence between the model and the data is small (see also Figure A7). Note we assume constant Domar weights in these aggregations. Bonadio et al. (2021b) studies the role of changing Domar weights over time. Throughout the quantitative analysis, we report correlations of growth rates. Appendix Table A6 shows that the properties of GDP growth rates to HP-filtered series for the G7 countries are quite similar.

Table 2: Correlated Shocks vs. Transmission Decomposition: G7 countries

	Mean	Median	25th pctile	75th pctile
		(N. obs. = 21)		
Data	0.358	0.333	0.122	0.552
Dynamic model	0.350	0.356	0.124	0.558
Decomposition in simulated model				
Shock Correlation	0.290	0.303	0.097	0.467
Bilateral Transmission	0.014	0.013	0.007	0.015
Multilateral Transmission	0.045	0.044	0.026	0.067

Notes: This table presents the decomposition of the GDP correlations into the shock correlation, the direct transmission, and the multilateral transmission terms as in equation (2.4).

Figure 2: Total Correlation, Shock Correlation, and Transmission, G7 Countries



Notes: This figure displays the network of GDP correlations (far left), decomposing it into the shock correlation (middle) and transmission (far right) components. Thicker lines denote higher values. Blue displays positive values, red negative values. Larger nodes (countries) displayed with bigger dots.

have a non-negligible contribution to the overall correlation, accounting for the remaining one-fifth.

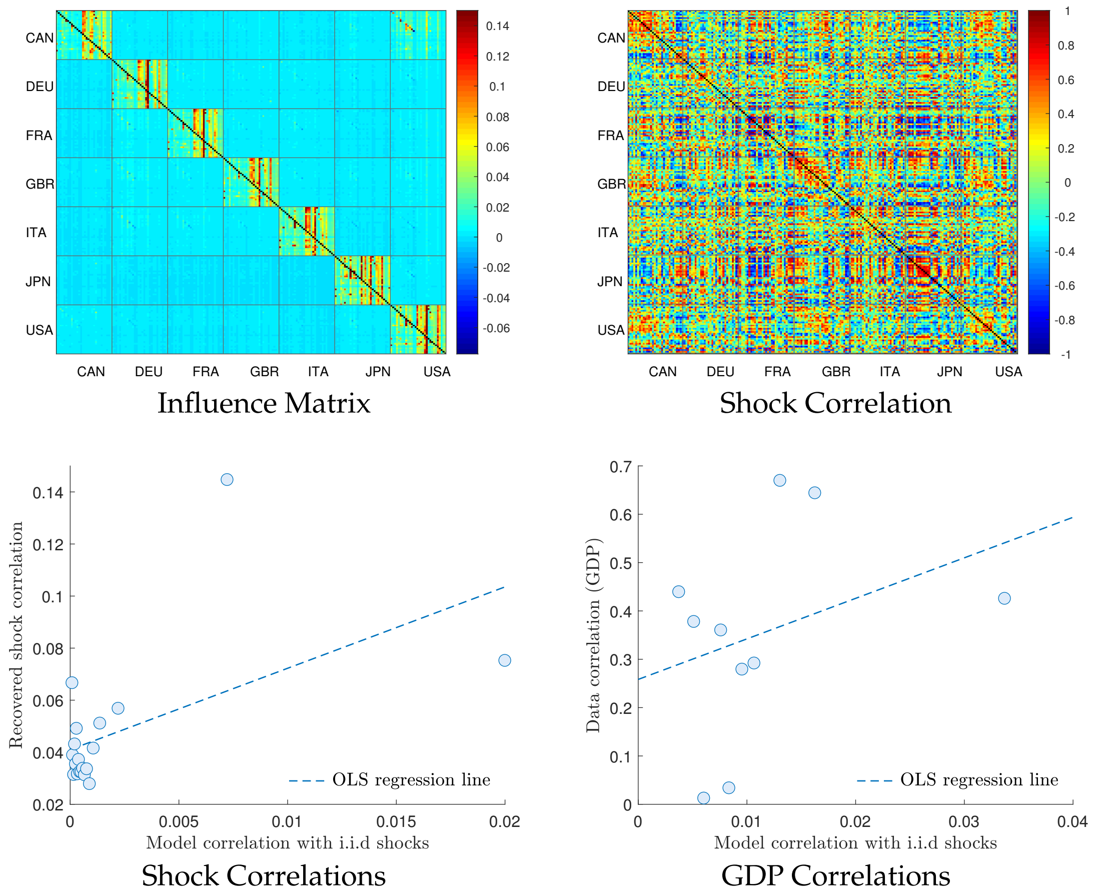
Figure 2 displays the heterogeneity in the correlation and transmission terms in a network graph for the G7 countries. The left panel depicts all the bilateral correlations among those countries, with thicker lines denoting larger values, and blue (resp. red) depicting positive (resp. negative) correlations. The middle panel displays the same for the shock correlation component, and the left panel the combined transmission. The scale (thickness of the lines) is the same in all three panels. It is clear that the differences in the shock correlation component are responsible for both the bulk of the overall correlation, as well as the variation across countries. For instance, none of the transmission components are negative, and thus all the negative actual correlations are due to the negative correlations of the shocks.

Influence Matrix and Shock Correlation. The top panel of Figure 3 displays two heat maps. In each, both rows and columns are broken into country-sectors, though due to space constraints sectors are too numerous to be labeled. In the top left is the influence matrix. It shares some clear similarities with the raw input-output and final shares matrices (Appendix Figure A2). Specifically, the largest positive entries tend to be domestic, and there are clear relationships between close trading partners like Canada and the US (upper right corner). However, there is one important difference: entries of the influence matrix are sometimes negative. Visible negative values in this heatmap are darker blue lines running parallel to the diagonal. These correspond to the same industries in different countries: in our influence matrix, a positive supply shock to foreign producers in the same industry tends to have a negative impact on sectoral output. This is in spite of the fact that often, the input shares in those sectors are also relatively high (the lines parallel to the diagonal are also evident in the input-output heat map). This discussion illustrates that the influence matrix conveys information distinct from the IO matrix itself.

The top right panel depicts the heat map of shock correlations. There is little if any similarity between this panel and the influence heat map. Visually, it does not even appear to be the case that within-country shock correlations are that much higher than the cross-country ones. As the overall GDP correlation is built from the shock correlation and the influence matrices, these two panels convey the sources of variation in these two components and the relative importance of the two. In particular, whereas the off-block-diagonal (cross-country) elements of the influence matrix by and large are both small and display limited variation, there is a great deal of variation in the cross-border shock correlation.

In order to better understand where and why the shock recovery procedure assigns correlated shocks, the bottom left panel of Figure 3 displays a binscatter of the the correlation of inferred shocks at the sector level against the sectoral value added correlations that arise when the model is subjected to i.i.d. shocks. The model with i.i.d. shocks produces correlation purely through transmission, as the shock correlation term is zero by construction. There is if anything a mildly positive relationship: the procedure assigns more positively correlated shocks in places where the model also generates the most transmission. However, the relationship is weak and driven mostly by the upper tail. Aggregating up to GDP, the bottom right panel displays a binscatter of the GDP correlations in the data on the y-axis against GDP correlations in the model subjected to i.i.d. shocks. According to Corollary 3.3, when the elasticities of substitution are around unity, the GDP correlation predicted by the model is positive. With our calibrated parameters, the implied relationship is also positive, but the model with i.i.d shocks does not generate the level of correlations observed in the data. Thus, the shock recovery procedure assigns a positive correlation to shocks because the internal propagation mechanisms are not sufficiently powerful to generate the observed levels of comovement. At the same time, the propagation mechanisms in the model go in the right direction in predicting comovement in the cross-section of sectors and countries. Thus, it is not the case that the assignment of shock

Figure 3: Input, Influence, and Correlation Heat Maps, G7



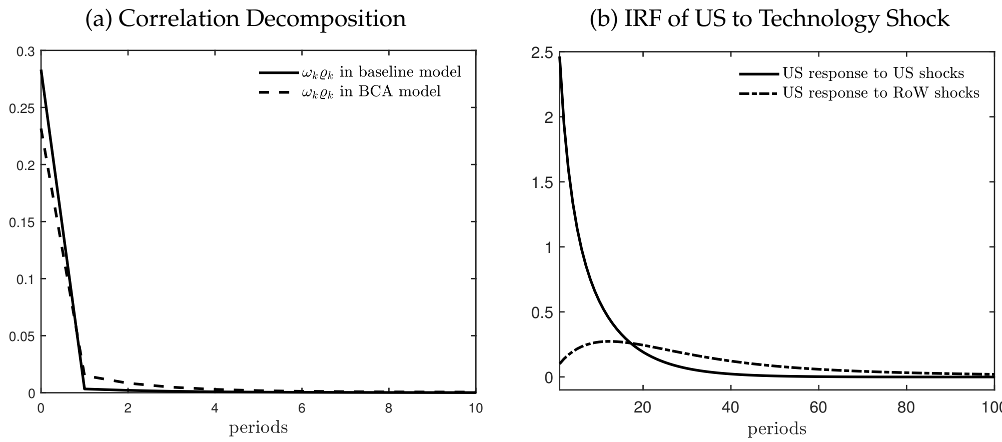
Notes: This figure displays the heat maps of the influence matrix (top left), the bilateral shock correlations (top right), the binscatter of the sector-pair correlations in the inferred shocks for 1978-2007 (y-axis) against correlations in value added in the model with i.i.d. shocks (x-axis) (bottom left), and the binscatter of the GDP correlations in the data (y-axis) against GDP correlations between country pairs in the model with i.i.d. shocks (x-axis) for the G7 countries (bottom right).

correlations has to "compensate" for the wrong predictions of the model regarding where higher and lower correlations should be.

Contemporaneous vs. Delayed Propagation. We next explore the quantitative importance of the intertemporal propagation relative to the contemporaneous response for international comovement. As emphasized in Section 2, in the dynamic model the GDP growth rate can be expressed as a function of current and past shocks (equation 2.2). Thus, the total GDP correlation is the weighted sum of the correlations of the responses to shocks at different horizons (equation 2.6).

The solid line in panel (a) of Figure 4 plots $\varrho_{nm,k}\omega_{nm,k}$ across horizons k, averaging over country pairs. The correlation of contemporaneous responses corresponds to $k = 0$. It turns out that the contemporaneous component is dominant, accounting for over 95% of the total correlation. That is,

Figure 4: Dynamic Correlation Decomposition



Notes: Panel (a) displays the elements of the dynamic decomposition of the overall correlation into the components accounted for by elements at horizon k, as in equation (2.6). The solid line displays the results for the baseline model in Section 4.5. The dashed line displays the results for the business cycle accounting model in Section 5.2. Panel (b) displays the impulse responses of US GDP following 1% shocks in all sectors in US (solid line), and following 1% shocks in the rest of the world excluding the US (dashed line).

adding dynamics does not substantially raise the GDP correlations.[27]

The model features rich intertemporal propagation patterns, as evidenced by Figure 1. The small contribution of dynamics to overall comovement is mainly due to the fact that the timing of the response to the same shock is not sufficiently similar across countries to induce substantial delayed correlation. To illustrate this pattern, panel (b) of Figure 4 compares the response of US GDP to its own and rest-of-the-world shocks. While the country responds positively and persistently to positive foreign shocks, both the shape and the magnitude of the responses are different. The response to its own shock is high on impact, and gradually dies out. In contrast, the response to other countries' shocks continues to build slowly and peaks after over twenty periods. In addition, the response to a country's own shock is far larger than the response to other countries' shocks, both on impact and at most horizons.

One reason why the model delivers so little delayed propagation is that the composite shocks $\ln Z_{nj,t}$ are not intertemporal. Looking ahead, Section 5.2 implements a business cycle accounting exercise with 4 shocks, one of which is intertemporal, acting on next period's capital. The dashed line in Figure 4 plots the dynamic decomposition in that model. Indeed, the role of delayed propagation in comovement is visibly greater in the presence of an intertemporal shock. However, it is still the case that the large majority of the total comovement comes from the impact responses.

[27]The overall correlation in Figure 4 is somewhat lower than the correlation reported in Table 2. To implement the decomposition (2.2), we must use the shock processes (4.2) and the analytical model solution rather than sequences of shock realizations over 1978-2007 as we do in Table 2. It turns out that the finite sample correlation over 1978-2007 is somewhat higher than the long-run correlation implied by the estimated shock processes.

The Sectoral Dimension. Appendix D.2 implements a decomposition similar to Section 2 at the sector level to complement this analysis and assess whether any sectors are especially influential in overall comovement. It turns out that services sectors such as real estate and wholesale trade sectors are the most prominent ones in contributing to comovement. Appendix D.3 fits some factor models to examine whether the recovered shocks exhibit a strong factor structure. While country and sector components are detectable in the panel of shocks, most of the variation is idiosyncratic.

4.6 Understanding Model Mechanisms

The analytical solution (3.13)-(3.14) makes it transparent that model quantification requires two sets of objects: (i) the input network (final and intermediate expenditure shares), and (ii) elasticities (of labor supply and substitution). To better understand the headline result, we perform exercises that highlight the roles of each of these.

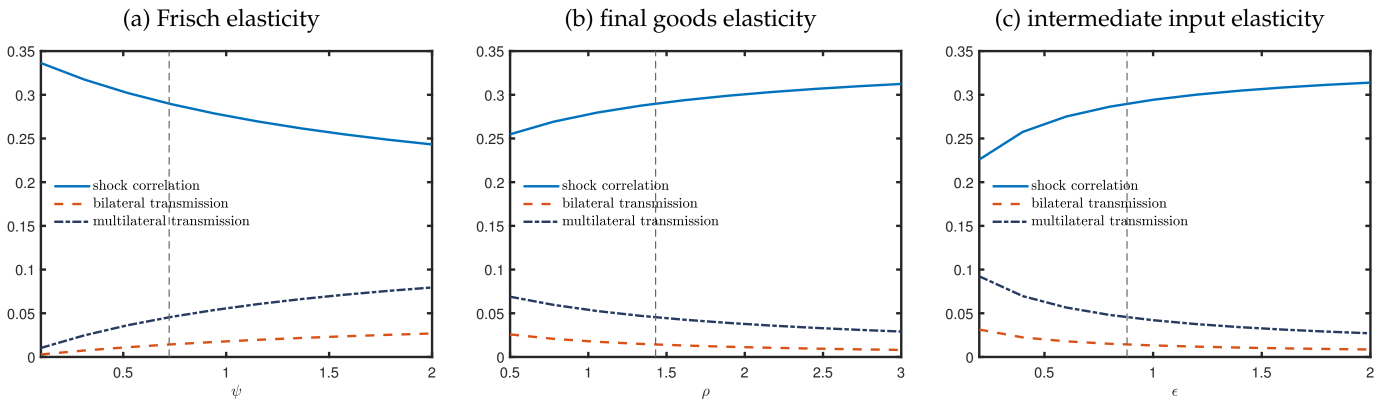
4.6.1 The Role of Elasticities

The model requires 3 elasticities: the Frisch elasticity of labor supply ψ, and substitution elasticities between inputs ε and final goods ρ. These elasticities govern the strength of the economy's response to foreign shocks, and therefore the extent of international shock transmission.

Intuitively, how much the domestic economy responds to a foreign shock depends on both demand for and supply of domestic value added. From the perspective of the domestic economy, a foreign productivity shock is a demand shock for its value added. The elasticity of substitution between domestic and foreign goods governs both the direction and the magnitude of the demand shift experienced by the home economy following a foreign productivity shock. The lower the elasticity, the more positive is the shift in demand for domestic goods. This is because with low elasticities, a fall in the price of one intermediate input strongly increases not just the quantity demanded of that input, but also quantity demanded of other inputs. The Leontief limit is particularly clear: when the price of a Leontief input falls, the using firm increases its demand for all inputs by the same proportional amount. When the elasticity of substitution goes above 1, demand for other inputs falls when an input becomes cheaper. The role of the low Armington elasticity in generating positive cross-country propagation has been highlighted since the early IRBC papers (Backus, Kehoe, and Kydland, 1992, 1995), and has been a consistent thread in the international macro literature since then (Heathcote and Perri, 2002; Kose and Yi, 2006; Burstein, Kurz, and Tesar, 2008; Corsetti, Dedola, and Leduc, 2008; Johnson, 2014, among many others).

Given a shift in demand, the Frisch elasticity then governs the magnitude of the domestic supply response. When the Frisch elasticity is low, factor inputs do not respond much to the shift in demand, muting the measured real GDP response. (The limiting case of perfectly inelastic factor supply is particularly clear, as a foreign productivity shock has no impact on domestic real GDP – there is no international transmission, see e.g. Kehoe and Ruhl, 2008.) Higher Frisch elasticities imply a larger

Figure 5: Correlated Shocks and Transmission, Alternative Elasticities



Notes: This figure displays the average contributions of correlated shocks and transmission in alternative calibrations of the baseline model. For each alternative parameter combination, we re-recover the shocks to match the observed sectoral value added. The vertical dashed line is the baseline calibrated value of that parameter.

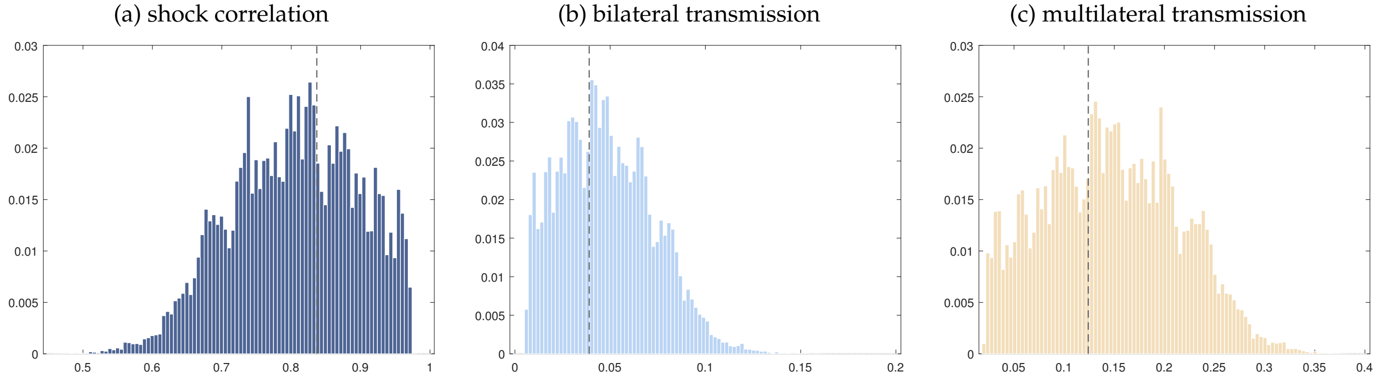
GDP response.

Figure 5 illustrates this discussion. The three panels plot the shares of shock correlation and of transmission in the total GDP comovement as a function of the three elasticities. In these figures, we re-do the shock extraction for each ψ, ε, and ρ. By construction, the procedure matches the data on the overall GDP correlations under each set of elasticities. However, since the shock extraction procedure itself relies on these elasticities, the fraction of comovement attributed to transmission varies with these parameters. Lower substitution elasticities and higher Frisch elasticities imply a higher share of transmission in total comovement, consistent with the intuition above.

Figure 5 makes it clear that the ultimate conclusion about the share of transmission in comovement depends on these parameters. As we estimate ε, ρ, and ψ, we can be explicit about the uncertainty surrounding these parameter values. Furthermore, estimation of ψ uses the values of ε and ρ, and thus is in that sense conditional on those. To provide a range of likely outcomes, we undertake the following simulation exercise. We draw ε and ρ from 1-standard-error intervals around their estimated values, and for each draw of a pair of (ε, ρ), we re-estimate ψ. Using the resulting triplet of elasticities, we then compute the shares of correlated shocks and transmission in the overall comovement. Repeating this simulation 1000 times gives us a distribution of these shares. Figure 6 reports the results as histograms of the shares of each component in overall comovement. The vertical dashed lines display the values under the point estimates. The simulation yields a range of outcomes. While some of the most extreme values imply that the share of transmission can be as high as 50%, those combinations of elasticities are unlikely. Most of the mass is in the range where shock correlation accounts for over 70% of the total comovement.[28]

[28]While our estimation strategy for ε and ρ is conventional and well-tested, estimating ψ in a quantitative international macro model is less common. Furthermore, the procedure uses the full structure of the model. As an alternative, we can

Figure 6: Distribution of Shares of Shock Correlation and Transmission



Notes: This figure displays the distributions of the shares of shock correlation (left panel), bilateral transmission (middle) and multilateral transmission (right) in average GDP correlations in the G7 countries. The elasticities of substitution ρ and ε are drawn from normal distributions with means/standard deviations corresponding to the point estimates/standard errors for ρ and ε in column 3 of Table A4, and the Frisch elasticity ψ is reestimated for each draw of a pair of ρ and ε.

4.6.2 The Role of the Input Network

To illustrate the role of the input network in generating the observed comovement, we compare the correlations in the baseline model to correlations that would obtain in an autarky counterfactual. Using the setup from Section 2, we can write the difference in GDP correlations between autarky and trade as a sum of two terms:

$$\Delta\rho_{nm} = \underbrace{\sum_{k=0}^{\infty}\sum_{j}\sum_{i}\left(\frac{s_{nnj,k}s_{mmi,k}}{\sigma_n\sigma_m} - \frac{s^A_{nj,k}s^A_{mi,k}}{\sigma^A_n\sigma^A_m}\right)\text{Cov}(\theta_{nj},\theta_{mi})}_{\Delta \text{ Shock Correlation}} \tag{4.3}$$
$$+ \text{ Bilateral Transmission} + \text{Multilateral Transmission},$$

where $s^A_{mi,k}$ are the elements of the influence vectors in autarky, and σ^A_n is the standard deviation of GDP growth in autarky.

Equation (4.3) shows that trade opening can affect the GDP correlation in two ways. First, it will make countries sensitive to foreign shocks, as captured by the Bilateral and Multilateral Transmission terms. Second, and more subtly, opening to trade will affect the economies' responses to their own domestic shocks: the domestic elements of the influence vectors will change. This is captured by the Δ Shock Correlation term. It will differ from zero when the shocks have some correlation,

draw from the distribution of ε and ρ, but treat ψ as a fixed ("calibrated") parameter, setting it to $\psi = 0.75$ as advocated by Chetty et al. (2011). Appendix Figure A6 displays the results. Not surprisingly, without variability in ψ the range of outcomes is narrower, with very few draws implying a share of transmission in total comovement of more than 20%.

$\text{Cov}(\theta_{nj}, \theta_{mi}) \neq 0$.[29]

Thus, in order to understand the contribution of international trade to international comovement, we must capture how going from autarky to trade changes the composition of the economy (and consequently the differences between $s_{nnj,k}$ and $s^A_{nj,k}$). We construct an autarkic economy by reassigning all final and intermediate expenditure shares on foreign goods to the same-sector domestically-produced goods. This is the most intuitive and parsimonious approach to construct autarky expenditure shares.[30] Using the resulting input and final consumption shares, we build the autarky influence matrix according to the same model solution (Section 3). We then apply the baseline shocks recovered in Section 4.3 to the autarky influence matrix to compute GDP growth rates in all countries and the resulting GDP correlations.

Panel (a) of Figure 7 displays the mean difference between the trade and the autarky correlations for each country, and decomposes it into the Δ Shock Correlation and transmission terms as in equation (4.3). Not surprisingly, moving from autarky to trade tends to increase the overall correlations (the blue bars), and the transmission terms contribute positively to this increase in comovement (yellow bars). Less expected, Δ Shock Correlation is actually negative for all countries. All else equal, when countries move from autarky to trade, their susceptibility to domestic shocks changes in such a way as to reduce overall comovement.

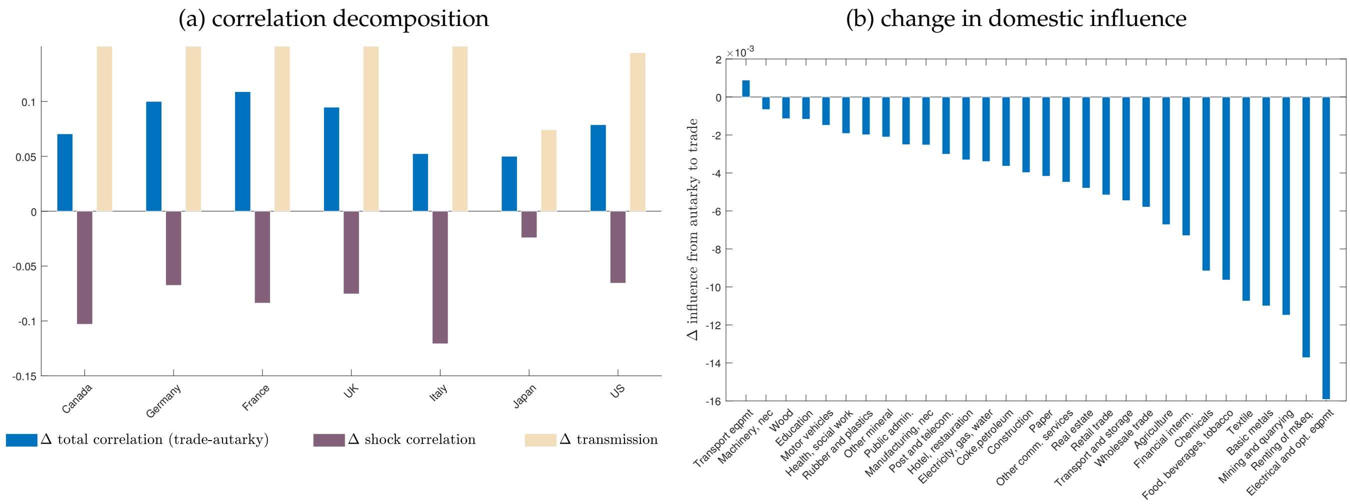
To understand this effect, panel (b) of Figure 7 plots the average changes in the domestic elements of the influence vectors when moving from autarky to trade, $s_{nnj,k} - s^A_{nj,k}$. On average, under trade $s_{nnj,k}$'s are lower than in autarky: economies are less susceptible to domestic shocks in an open economy compared to the closed one. The shrinking of influence is most pronounced in tradeable sectors: 6 of the top 7 largest average reductions in influence are in manufacturing and mining. Intuitively, when a country sources a significant share of its sectoral expenditure abroad, domestic sectoral conditions matter less and domestic shocks propagate less to GDP. As a result, it matters less that domestic shocks are positively correlated across countries. Of course, this is a *ceteris paribus* effect. Domestic influence is replaced by foreign influence when countries open to trade, and comovement increases overall. But it increases by less than partial equilibrium reasoning would imply.

All in all, this exercise shows that in counterfactual exercises with respect to the role of international trade, we must keep track of the entire reshuffling of the production network, including importantly the domestic linkages. This lesson is relevant, for example, in analyzing the effects of renationalizing supply chains as in Bonadio et al. (2021a). Appendix D.4 presents the results of two intermediate

[29] Because we are working with correlations, changes in the the standard deviation of GDP between autarky and trade, σ^A_n vs. σ_n, also appear in this expression, but this is not a large effect in practice.

[30] A natural way to construct autarky would be to set the trade costs to infinity. Because the substitution elasticity between inputs ε is less than 1, raising τ_{mni} will actually *increase* expenditure shares on imports. This value of ε is meant to govern the short-run responses of the economy to shocks, and is estimated from year-to-year variation. It is clearly not the right elasticity to apply to "long-run" structural changes in the economy like opening to trade (see Boehm, Levchenko, and Pandalai-Nayar, 2022, for empirical evidence that long-run elasticities are larger than short-run elasticities). Our autarky counterfactual can be conceived as a limiting case as all $\tau_{mnj} \to \infty$ while the substitution elasticity between inputs approaches 1 from above: $\varepsilon \downarrow 1$.

Figure 7: Change in GDP Correlation between Trade and Autarky: Decomposition



Notes: Panel (a) displays the mean change in the GDP correlation going from autarky to trade (blue bars), decomposing it into the mean ΔShock Correlation$_{mn}$ (brown bars) and transmission terms (yellow bars). Panel (b) displays the mean change, by sector, in the domestic terms of the influence vector under trade relative to autarky, $s_{nnj,k} - s^A_{nj,k}$.

counterfactuals, in which we shut down intermediate input and final goods trade one at a time. As expected, intermediate input trade has a stronger synchronizing impact than final goods trade. Appendix D.5 relates our results to the well-known trade-comovement regressions. It shows that much of the overall trade-comovement slope is in fact due to the positive relationship between the Shock Correlation term and trade intensity, echoing Imbs (2004).

5. Extensions

5.1 Full Country Sample and the Static Network Model

For all the 29 countries in the sample, we lack sufficient time series data to estimate the stochastic process for the shocks required to implement the dynamic model. However, we can still work with a static network version of the model, described in Section 3.2. In this model, the shock recovery procedure is a matrix inversion that takes place in one step, as discussed in Section 4.3. Table 3 reports the results. The top panel summarizes the G7 correlations, to be compared with Table 2. The middle panel reports the results for all countries. Two conclusions emerge. First, for the G7 the static and dynamic models give essentially the same answers on the share of overall comovement accounted for by the different components. Second, in the full sample correlations are lower on average, but the share accounted for by transmission is very similar to the G7.

Table 3: Correlated Shocks vs. Transmission Decomposition, Extensions

	Mean	Median	25th pctile	75th pctile
Static Network Model, G-7 countries (N. obs. = 21)				
Data	0.358	0.333	0.122	0.552
Static network model	0.350	0.356	0.124	0.558
Decomposition				
Shock Correlation	0.293	0.298	0.102	0.499
Bilateral Transmission	0.014	0.011	0.006	0.016
Multilateral Transmission	0.043	0.047	0.022	0.056
Static Network Model, All countries (N. obs. = 406)				
Data	0.187	0.183	-0.110	0.500
Static network model	0.193	0.198	-0.104	0.498
Decomposition				
Shock Correlation	0.149	0.179	-0.122	0.431
Bilateral Transmission	0.005	0.003	0.001	0.006
Multilateral Transmission	0.039	0.029	0.011	0.066
Four-Shock Model, G-7 countries (N. obs. = 21)				
Data	0.358	0.333	0.122	0.552
4-shock dynamic model	0.350	0.356	0.124	0.558
Decomposition				
Shock Correlation	0.290	0.290	0.106	0.472
Bilateral Transmission	0.050	0.045	0.033	0.057
Multilateral Transmission	0.010	0.007	-0.017	0.039

Notes: This table presents the decomposition of the GDP correlations into the shock correlation, the direct transmission, and the multilateral transmission terms as in equation (2.4) in the static network model for G7 and all countries, and in the dynamic model with 4 shocks.

5.2 Multiple Shocks

The composite sectoral shock extracted in Section 4 subsumes changes in productivity, as well as changes in labor and capital inputs that the model cannot produce endogenously. This composite shock has two potential limitations. The first is that it is by construction not especially informative on the deeper underlying drivers of business cycles in general, and of international comovement in particular. Second, at a more technical level, only the intratemporal shocks to primary factor supplies can be made isomorphic to TFP, $\mathbf{Z}_t$. Intertemporal shocks cannot be extracted using this approach. Shocks to frictions in the intermediate input market are also not isomorphic to TFP. Thus, if shocks to the intermediate input market are non-trivial, the procedure in Section 4.3 would recover a supply shock that is in part a linear combination of all countries' and sectors' intermediate input market shocks.

This section enriches the model to allow for several distinct shocks. One benefit of this extension is that we can now perfectly match multiple series: value added, labor, capital, and intermediate inputs. We accommodate intertemporal as well as intratemporal shocks, and thus we can quantify their relative importance.

The household problem is now subject to 2 shocks: labor ξ^H_{njt} and investment ξ^I_{njt}, which we interpret as distortive taxes following Chari, Kehoe, and McGrattan (2007).[31] The budget constraint becomes:

$$P_{n,t}\left(C_{n,t}+\sum_j\left(1+\xi^I_{nj,t}\right)I_{nj,t}\right)=\sum_j\left(1-\xi^H_{nj,t}\right)W_{nj,t}H_{nj,t}+\sum_j R_{nj,t}K_{nj,t}+T_{n,t}$$

where $T_{n,t}$ is a lump-sum transfer that rebates to the households all the within-country taxes. The labor supply and Euler equations now read:[32]

$$H^{\frac{1}{\psi}}_{nj,t}=\left(1-\xi^H_{nj,t}\right)\frac{W_{nj,t}}{P_{n,t}}, \tag{5.1}$$

and

$$U'_{n,t}\left(1+\xi^I_{nj,t}\right)=\beta\mathbb{E}_t\left[U'_{n,t+1}\left(\frac{R_{nj,t+1}}{P_{n,t+1}}+(1-\delta)\left(1+\xi^I_{nj,t+1}\right)\right)\right].$$

While capital accumulation does not feature explicit adjustment costs, Chari, Kehoe, and McGrattan (2007) point out that the investment shock plays much the same role as adjustment costs.

Production is subject to a TFP shock, which we relabel $\xi^Z_{nj,t}$ in this section,[33] and a shock to the

[31] Note that modeling these wedges as shocks to preferences and technologies or distortive taxes does not matter for accounting purposes or the primary question in this paper, but will have different welfare implications (Chari, Kehoe, and McGrattan, 2009).

[32] The labor shocks can have a literal interpretation as exogenous shifts in intratemporal factor supply curves. Alternatively, news shocks (e.g. Beaudry and Portier, 2006), or sentiment shocks (e.g. Angeletos and La'O, 2013; Huo and Takayama, 2015) would manifest themselves as shocks to $\xi^H_{nj,t}$, as agents react to a positive innovation in sentiment by supplying more labor. Straightforward manipulation shows that $\xi^H_{nj,t}$ can also be viewed as a shifter in the optimality condition for factor usage. The literature has explored the aggregate labor version of this shifter, labeling it alternatively a "preference shifter" (Hall, 1997), "inefficiency gap" (Galí, Gertler, and López-Salido, 2007), or "labor wedge" (Chari, Kehoe, and McGrattan, 2007). While this object is treated as a reduced-form residual in much of this literature, we know that monetary policy shocks under sticky wages (Galí, Gertler, and López-Salido, 2007; Chari, Kehoe, and McGrattan, 2007), or shocks to working capital constraints (e.g. Neumeyer and Perri, 2005; Mendoza, 2010) manifest themselves as shocks to $\xi^H_{nj,t}$.

[33] The $\xi^Z_{nj,t}$ match the measured Solow residual, and differ from the Z shock recovered in Section 4.

intermediate input market $\xi^X_{nj,t}$. The firm thus solves:

$$\max \left\{ P_{nj,t} Y_{nj,t} - W_{nj,t} H_{nj,t} - R_{nj,t} K_{nj,t} - (1 + \xi^X_{nj,t}) P^x_{nj,t} X_{nj,t} \right\}$$

where

$$Y_{nj,t} = \xi^Z_{nj,t} \left(K^{1-\alpha_j}_{nj,t} H^{\alpha_j}_{nj,t} \right)^{\eta_j} X^{1-\eta_j}_{nj,t}. \tag{5.2}$$

The intermediate goods shock affects the input choice decision:

$$(1 - \eta_j) P_{nj,t} Y_{nj,t} = (1 + \xi^X_{nj,t}) P^X_{nj,t} X_{nj,t}. \tag{5.3}$$

The market clearing conditions are unchanged, and still given by (3.11).

Recovering Shocks. As mentioned above, with 4 shocks instead of 1, we match multiple data series: value added, labor input, capital input, and intermediate input. Three of the 4 shocks can be recovered by applying data for $Y_{nj,t}$, $H_{nj,t}$, $K_{nj,t}$, and $X_{nj,t}$ to intratemporal optimality conditions, and thus do not rely on the dynamic structure of the model. The TFP shock is read simply as the Solow residual from (5.2). The labor shock is recovered from the labor supply (5.1), after expressing $W_{nj,t}$ and $P_{n,t}$ as functions of $Y_{nj,t}$ and model parameters. The intermediate shock $\xi^X_{nj,t}$ comes from (5.3), after expressing the prices as functions of $Y_{nj,t}$.

The TFP shock is the least reliant on model parameters, as it needs information only on factor shares α_j and η_j. The intermediate market shock requires in addition substitution elasticities ρ and ε, plus the global matrix of input and final good shares. Recovery of the labor shocks relies on all of those plus the Frisch elasticity.

The investment shock $\xi^I_{nj,t}$ enters the Euler equation, and thus requires solving the full dynamic model. To do that requires specifying the stochastic processes for all 4 shocks, as policy functions depend on the perceived stochastic shock processes. In exercises similar to ours, Chari, Kehoe, and McGrattan (2007) and Ohanian, Restrepo-Echavarria, and Wright (2018) use maximum likelihood and Bayesian estimation respectively to obtain the shock processes. However, the number of parameters to be estimated in our exercise is an order of magnitude larger because all shocks are at the sectoral level, which makes either maximum likelihood or Bayesian estimation intractable. Instead, we follow an approach similar to the composite shock recovery procedure in Section 4.3. We impose parsimonious functional forms on the shock processes:

$$\ln \xi^x_{nj,t} = \rho^x_{nj} \ln \xi^x_{nj,t-1} + \zeta^x_n \mathbf{1}\left(m = n, k \neq j\right) \ln \xi^x_{mk,t-1} + \theta^x_{nj,t} \tag{5.4}$$

for $x = Z, X, H, I$. We then apply an iterative procedure to recover the shock processes. Given some perceived shock processes, we compute the policy functions. Under these candidate policy functions, we can recover the time series for the realizations of shocks that rationalize the data. These candidate shock realizations can in turn be used to estimate the stochastic shock processes, which become the perceived processes in the next iteration. We iterate until the perceived and estimated shock processes coincide. Appendix Table A5 summarizes the parameter estimates for the processes. The estimated processes are persistent, with mean own-lag parameter estimates ranging from 0.85 for $\ln \xi^X_{nj,t}$ to 0.95 for $\ln \xi^I_{nj,t}$. Lagged cross-sector within-country spillovers are very small and close to zero on average for all series.

The bottom panel of Table 3 reports the main decomposition (2.4) of the overall comovement into correlated shocks and transmission. The extension of the decomposition to 4 shocks is straightforward. The main result on the relative importance of correlated shocks vs. transmission is virtually the same in the 4-shock model compared to the baseline.

The Role of Individual Shocks. A well-known feature of this type of business cycle accounting exercise is that the 4 extracted shocks are not mutually uncorrelated (see, among others, Chari, Kehoe, and McGrattan, 2007; Eaton et al., 2016). Thus, part of the GDP correlation will come from cross-shock covariance terms, for instance comovement driven by correlation of TFP in country j with the labor shock in country i.[34]

We proceed by presenting two polar exercises. In the first, we take out one shock at a time, keeping the other 3. In terms of shock correlation, removing a shock gets rid of both the correlations of that shock with the same shock in other countries (e.g., TFP in country j with TFP in country i), and the correlation of that shock with other shocks in other countries (e.g., TFP in country j with labor in country i). In the second exercise, we feed in one shock at a time. This exercise generates comovement only through correlation of a shock with the same shock abroad (in addition to transmission). Needless to say, in both types of exercises, the transmission terms change as well. Note that shocks can have a negative correlation, and thus it is not necessarily the case that the first exercise leads to a larger contribution of a single shock to comovement compared to the second exercise.

Table 4 reports the resulting correlations. The first two rows present the data and the correlations conditional on all four shocks, which by definition match the data. The next 4 rows remove one shock at a time. The largest impact on correlation is due to the labor shock: removing it lowers the mean correlation by nearly 60%. Removing the other shocks has much less impact, conditional on the other shocks operating.

The bottom four rows report instead the correlations conditional on a single shock. The correlation generated by the labor shock, 0.24, is more than double the 0.11 correlation generated by the TFP shocks. Interestingly, the intermediate input shocks and especially the investment shocks by themselves itself generates the highest correlations. The two rightmost columns of Table 4 report two additional diagnostics on the alternative shock models: the standard deviation of GDP growth, and

[34]As in other business cycle accounting analyses, different types shocks within the same country are correlated among themselves. With a small number of aggregate shocks in a single country, one can in principle orthogonalize them. Our object is international correlations, and there are 4 shocks in each sector in each country. There is no practical way to transform them in such a way that a given type of shocks in one country is only correlated with the same type of shocks in the other countries, but orthogonal to all other categories of foreign shocks.

Table 4: GDP Growth Correlations Conditional on Subsets of Shocks

	Cross-country correlation			
	Mean	Median	St.dev.(GDP)	Corr w/data
Data	0.358	0.333	1.667	1.000
All shocks	0.350	0.356	1.663	1.000
No TFP shock	0.357	0.337	1.297	0.530
No labor shock	0.142	0.169	1.560	0.830
No intermediate input shock	0.371	0.353	1.689	0.977
No investment shock	0.403	0.376	0.703	0.478
Only TFP shock	0.103	0.090	1.528	0.663
Only labor shock	0.235	0.197	0.914	0.399
Only intermediate input shock	0.283	0.184	0.367	0.028
Only investment shock	0.599	0.574	0.711	0.539
TFP and labor	0.273	0.333	1.492	0.901

Notes: This table presents the summary statistics of the correlations of $d \ln G_{n,t}$ in the sample of G7 countries driven by various shocks.

the correlation between GDP growth generated by each subset of shocks and the data GDP growth. The intermediate and investment shocks do not generate sufficient volatility. Alone, the intermediate input and investment shocks produce standard deviations of GDP growth that are only 20 and 40% of the data values, respectively. GDP growth driven by intermediate input shock is uncorrelated with the data. By contrast, while the TFP shock by itself does not produce a lot of international comovement, the GDP growth rates generated by TFP are the most correlated with the data compared to those generated by other shocks, as evidenced by the last column of Table 4.

It may appear puzzling that the labor shock comes out as the most important in the first exercise, but less so in the second one. The discrepancy in the conclusions from the two exercises is resolved by the fact that the labor shock is positively correlated with other shocks abroad, whereas the investment and intermediate shocks are negatively correlated with other shocks abroad. Table 5 illustrates this by reporting the mean GDP correlations when GDP in country j is driven by a shock in the row of the table, and GDP in country i is driven by a shock in the column. The labor shock-driven GDP is positively correlated with the TFP-driven GDP (0.126), and with the investment shock-driven GDP (0.242). By contrast, the investment and intermediate shock-driven GDP has negative correlations with other shocks.

When shocks are mutually correlated, one way to decompose the overall comovement into the

Table 5: Correlations Among GDP Growth Rates Driven by Single Shocks

	TFP	Labor	Intermediate	Investment	Shapley Value (share of total)
TFP	0.103	0.025	0.196	-0.055	0.271
Labor	0.124	0.235	-0.069	0.250	0.429
Intermediate	0.093	-0.047	0.283	-0.189	-0.042
Investment	-0.023	0.284	-0.130	0.599	0.342

Notes: The first 4 columns of this table reports the average GDP correlations that result when GDP growth in one country is driven by the shock in the row of the table, and the GDP growth in the other country is driven by the shock in the column. The last column reports the Shapley value of each shock, expressed as a share of the total comovement, in contributing to the average GDP covariance in the data.

contributions of individual shocks is the Shapley (1953) value approach. Essentially, the Shapley value averages the contribution to the overall comovement of each shock across all the possible permutations of the 4 shocks. It answers the question of how much comovement increases on average when a shock gets added to the model. Expressed as shares of the total comovement, the contributions of all 4 shocks add up to 1. One challenge with this exercise is that different shocks produce very different business cycle volatilities (Table 4). As a result, the GDP correlations they yield are not necessarily directly comparable. Thus, we carry out the Shapley value decomposition on the average covariances instead of correlations. The last column of Table 5 reports the results. The labor shock is the single most important shock according to this metric, followed by the investment and the TFP shocks that are of roughly equal importance. The intermediate shock's Shapley value is close to zero.[35]

We synthesize these results as follows. First, no single shock has a dominant role in international comovement. Individually, the labor and the TFP shocks appear most promising, but for different reasons. The labor shock has the highest synchronizing impact, with the qualification that much of its overall effect appears to come from its correlation with other shocks rather than with itself. Taken alone, the TFP shocks generate less GDP comovement, but they produce GDP series closer to the data, as measured by both standard deviation and correlation. On the face of it, intermediate and investment shocks generate the most correlation alone, but that comes in part due to their negative correlations with other shocks. As a result, the GDP series generated by these shocks alone do not look much like the data.

Second, a model that combines labor and TFP shocks strikes a good balance between parsimony and fit to the data. The bottom row of Table 4 reports the statistics for the model performance with these two shocks. The two shocks together generate 80% of the observed international correlation. At the same time, they reproduce the observed GDP volatility and generate a GDP series with an

[35]To further illustrate the properties of the different shocks, Appendix D.7 projects the extracted shocks on a vector of shocks identified elsewhere in the literature, and presents a historical narrative that attributes changes in international GDP correlations to changes in the realizations of different shocks over time.

0.90 correlation with the data. The model is parsimonious both in the sense that it relies on only two shocks, and in the sense that these shocks themselves are relatively simple, and would work in the same way in both static and dynamic models.

Third, the next shock in order of importance is the investment shock. Adding it to TFP and labor essentially reproduces the data (see row "No intermediate input shock" in Table 4). However, adding this shock comes at a cost of parsimony, especially because both extracting and using this shock requires solving and iterating on the full dynamic model. While the baseline model abstracts from modeling financial shocks explicitly, as shown by Chari, Kehoe, and McGrattan (2007) some types of financial frictions will manifest themselves as investment shocks. Our results can be viewed as suggestive that financial shocks may play a non-trivial role in international comovement, but perhaps a less important one than the TFP and labor shocks.

Finally, the intermediate input shock is the least successful, either by itself or in conjunction with other shocks. A quantification is not missing much by omitting it.

5.3 Rigidities in Labor Markets

The baseline analysis is carried out under flexible goods and factor prices. We do not consider goods price stickiness, as that would require a departure from perfect competition in the goods markets. This subsection extends the baseline one-shock model from Sections 3-4 to incorporate wage rigidities, which are pervasive and persistent in the data (Schmitt-Grohé and Uribe, 2016; Grigsby, Hurst, and Yildirmaz, 2021).

To make the analysis more comparable with our baseline framework and to avoid specifying the monetary policy rules in multiple countries, we assume that wages are rigid in real terms. Denote by $w_{nj,t} \equiv \frac{W_{nj,t}}{P_{n,t}}$ the real wage in country n and sector j. We make two assumptions. First, the law of motion (in percentage deviations from the steady state) is given by

$$\ln w_{nj,t} = (1-\lambda)\ln w^*_{nj,t} + \lambda w_{nj,t-1}, \tag{5.5}$$

where $w^*_{nj,t}$ is the real wage in the absence of wage rigidities, and λ parameterizes the degree of wage stickiness. When $\lambda = 0$, we return to the baseline model in Section 3. This specification captures the partial dependence on past wage levels, and is similar in spirit to the modeling of non-optimizing backward-looking firms in Fuhrer and Moore (1995) and Galí and Gertler (1999). Compared with the wage Philips curve with Calvo-type wage rigidities, condition (5.5) abstracts from the forward-looking wage determinants. Second, such frictions necessarily prevent labor markets from clearing. Following the literature on business cycle models subject to nominal rigidities, we assume that the outcomes in the labor market are demand determined. That is, workers do not always operate on their intratemporal optimality conditions.

In this setting, following a higher productivity or lower intermediate goods prices, the lack of upward adjustment in wages results in a larger increase the quantity of labor demanded. This manifests itself in labor market outcomes being more responsive, which is as if workers had a higher Frisch elasticity in a model without nominal rigidities. The following proposition formalizes this argument.

Proposition 5.1. *When real wages are governed by* (5.5)*, the impact response of output to a vector of shocks* $\ln \mathbf{Z}_t$ *is*

$$\ln \mathbf{Y}_t = \mathbf{\Lambda}(\infty)\left[\mathbf{\Lambda}\left(\frac{\psi + 1 - \lambda}{\lambda}\right)\right]^{-1}\mathbf{\Lambda}(\psi)\ln \mathbf{Z}_t,$$

where $\mathbf{\Lambda}(x)$ *is the impact response of the flexible-wage* ($\lambda = 0$) *economy with Frisch elasticity* x*.*

Proof. See Appendix B. ◻

Note that when λ approaches zero, the influence matrix returns to $\mathbf{\Lambda}(\psi)$ which is the baseline case in the absence of wage rigidities. When λ approaches one, the influence matrix becomes $\mathbf{\Lambda}(\infty)$, implying that the sticky wage model with $\lambda = 1$ is isomorphic to the baseline model with an infinitely high Frisch elasticity. This proposition illustrates that as the wage rigidity becomes more severe, the force of internal propagation strengthens and the bilateral and multilateral transmission are more important. This property is consistent with the findings of Ho, Sarte, and Schwartzman (2022) under price rigidity.

Figure 8 displays the international comovements decomposition (2.4) as a function of λ. Our baseline model ($\lambda = 0$) remains a good approximation for economies with the duration of real wages are less than 5 years ($\lambda \leq 0.2$). Even for values of λ as high as 0.5, shock correlation accounts for over 50% of overall comovement.

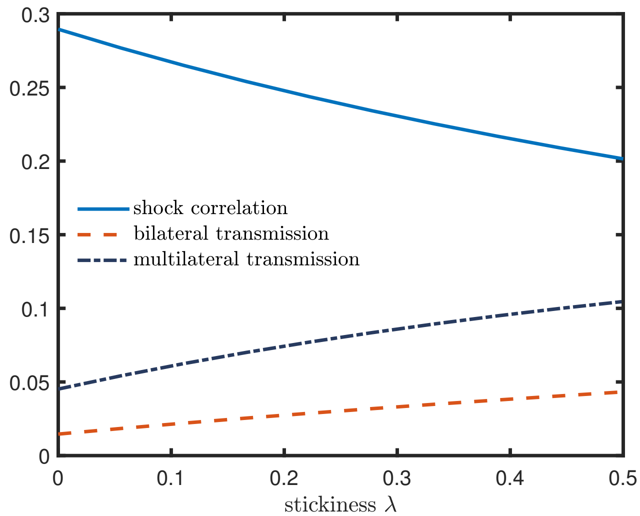
Financial integration. Appendix D.6 extends the model to allow countries to trade a complete set of state-contingent securities. The main conclusions about the relative importance of correlated shocks vs. transmission are robust to this extension.

6. Conclusion

We set out to provide a comprehensive account of international comovement in real GDP. Using a simple accounting framework, we decomposed the GDP covariance into additive components representing correlated shocks and cross-border transmission. The relative importance of these two terms is determined jointly by the correlations of the primitive shocks and the strength of domestic and international input-output linkages. The accounting framework also clarifies the role of dynamic propagation: the total GDP correlation is the sum of the correlation due to the instantaneous responses to shock innovations, and dynamic terms that capture the lagged responses to shocks.

While transmission of shocks has an economically meaningful role, most of the observed GDP comovement is accounted for by correlated shocks. The majority of the observed overall correlation

Figure 8: Correlated Shocks and Transmission with Wage Rigidity



Notes: This figure displays the average contributions of correlated shocks and transmission in models with different degrees of wage rigidities.

is due to the instantaneous response of the economy to shocks, rather than dynamic propagation of past shocks. And finally, while no single shock is predominantly responsible for international comovement, a relatively parsimonious model with two shocks – TFP and labor – appears to generate the bulk of the observed GDP correlations.

Our results suggest that when searching for correlated shocks that synchronize GDP, Solow residual shocks are not sufficient, and that we should instead focus on non-technology shocks that have a labor wedge representation in a prototype model. Structural estimates of various candidate shocks that might explain the correlation in these "labor wedges" across countries are not yet available, and identifying them is a promising avenue for future research. Bui et al. (2022) take a step in this direction by modeling and measuring informational frictions in the global value chains, that can give rise to international sentiment shocks. A related finding is that introducing wage rigidities increases the relative importance of transmission in international comovement. Thus, the role of rigidities should also be explored in future research (e.g. Ho, Sarte, and Schwartzman, 2022).

Beyond the specific substantive focus of this paper, we have provided a conceptual and quantitative framework to study international shock transmission through input networks. This framework remains tractable under a variety of extensions, and can readily be used in a variety of further applications by future researchers.

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ONLINE APPENDIX
(NOT FOR PUBLICATION)

Table of Contents

A. Data Description

Tables A1 and A2 list the sectors and countries in our analysis. Figure A1 illustrates the average imported input intensity by sector in our data. Figure A2 displays two heat maps for the G7 countries. In each, both rows and columns are broken into country-sectors, though due to space constraints sectors are too numerous to be labeled. On the left is the usual heat map of log intermediate input shares in the WIOD, with the suppliers on the x-axis, and input users on the y-axis. Versions of this heat map have appeared in the literature (see, e.g., Jones, 2013). The most saturated reds, indicating greater input linkages, are in blocks on the diagonal, corresponding to countries' domestic linkages. On the right is the heat map of log final expenditure shares instead. For final shares, there is no notion of a using (y-axis) sector, as final expenditure is undertaken by a representative consumer in each country. Nonetheless, we keep the format of the plot the same. Once again, domestic final shares are the highest, but there is meaningful variation across sectors within country pairs. Both intermediate and final shares are inputs into the influence matrix.

Table A1: Sectors

agriculture hunting forestry and fishing	basic metals and fabricated metal
financial intermediation	mining and quarrying
machinery nec	real estate activities
food beverages and tobacco	electrical and optical equipment
renting of m&eq and other business activities	textiles textile leather and footwear
transport equipment	public admin and defense; compulsory social security
wood and of wood and cork	manufacturing nec; recycling
education	pulp paper paper printing and publishing
electricity gas and water supply	health and social work
coke refined petroleum and nuclear fuel	construction
other community social and personal services	chemicals and chemical products
hotels and restaurants	sale maintenance and repair of motor vehicles
rubber and plastics	transport and storage
wholesale trade and commission trade	other nonmetallic mineral
post and telecommunications	retail trade except of motor vehicles

Table A2: Country Sample

G7	Rest			
Canada	Australia	Finland	Lithuania	Spain
France	Austria	Greece	Netherlands	Sweden
Germany	Belgium	Hungary	Poland	
Italy	Cyprus	India	Portugal	
Japan	Czech Republic	Ireland	Russian Federation	
UK	Denmark	Republic of Korea	Slovak Republic	
USA	Estonia	Latvia	Slovenia	

Figure A1: Average Sectoral Imported Input Intensity

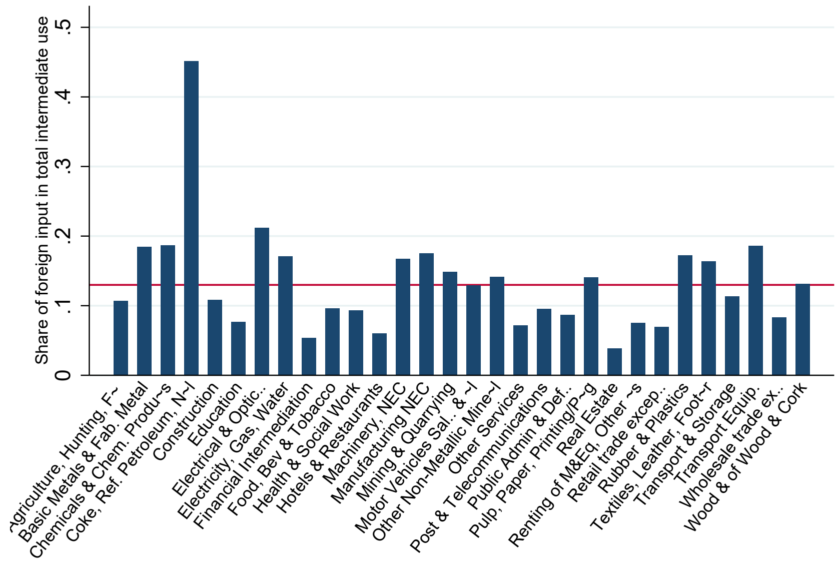
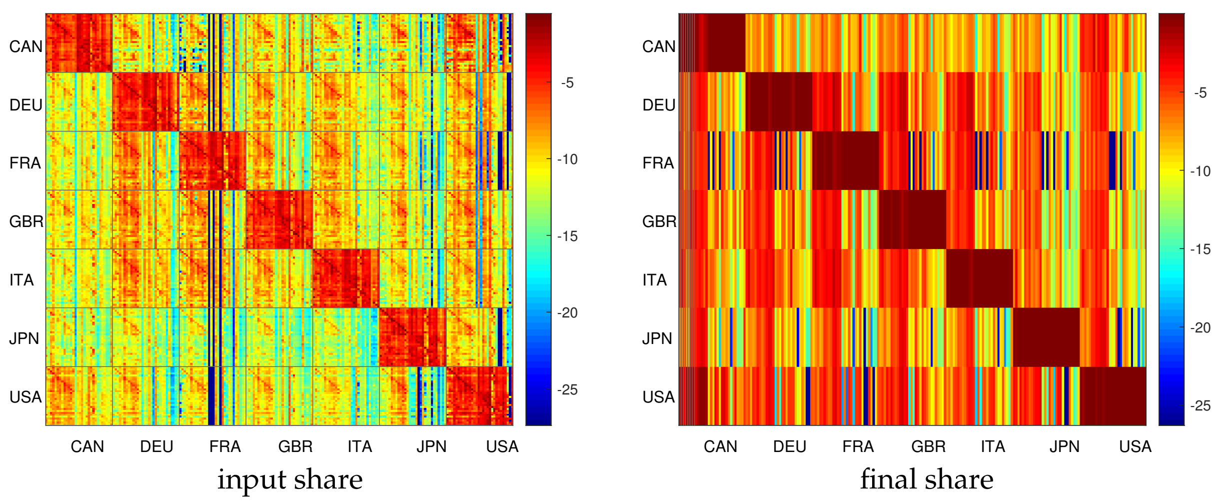


Figure A2: Input and Final Use Heat Maps, G7



Notes: This figure displays the heat maps of the log intermediate input shares in WIOD (left) and the log final spending shares (right) for the G7 countries.

B. Model and Quantitative Results

Throughout this appendix, variables without a t-subscript denote steady-state values, and variables with a t-subscript denote their realizations following a shock.

B.1 Proofs

Proof of Proposition 3.1:. The market clearing condition for the sales in country n sector j in levels is

$$P_{njt}Y_{nj,t} = \sum_m P_{m,t}\mathcal{F}_{m,t}\pi^f_{nmj,t} + \sum_m \sum_i (1-\eta_i)P_{mi,t}Y_{mi,t}\pi^x_{nj,mi,t}.$$

Note that with financial autarky, the total sales of final goods is the same as the value added across sectors

$$P_{m,t}\mathcal{F}_{m,t} = \sum_i \eta_i P_{mi,t}Y_{mi,t}.$$

The market clearing condition is then

$$P_{nj,t}Y_{nj,t} = \sum_m \sum_i \eta_i P_{mi,t}Y_{mi,t}\pi^f_{nmj,t} + \sum_m \sum_i (1-\eta_i)P_{mi,t}Y_{mi,t}\pi^x_{nj,mi,t}.$$

Log-linearizing:

$$\begin{aligned}\ln P_{nj,t} + \ln Y_{nj,t} = & \sum_m \sum_i \frac{\pi^f_{nmj}P_m\mathcal{F}_m}{P_{nj}Y_{nj}} \frac{\eta_i P_{mi}Y_{mi}}{P_m\mathcal{F}_m}\left(\ln P_{mi,t} + \ln Y_{mi,t} + \ln \pi^f_{nmj,t}\right) \\ & + \sum_m \sum_i \frac{(1-\eta_i)\pi^x_{nj,mi}P_{mi}Y_{mi}}{P_{nj}Y_{nj}}(\ln P_{mi,t} + \ln Y_{mi,t} + \ln \pi^x_{nj,mi,t}) \end{aligned} \tag{B.1}$$

and the log-deviation of import shares are given by

$$\begin{aligned}\ln \pi^f_{nmj,t} =& (1-\rho)\sum_k \pi^c_{kmj}\left(\ln P_{nj,t} - \ln P_{kj,t}\right) \\ \ln \pi^x_{nj,mi,t} =& (1-\varepsilon)\sum_{k,\ell} \pi^x_{kl,mi}\left(\ln P_{nj,t} - \ln P_{k\ell,t}\right).\end{aligned}$$

Define the following share matrices:

1. $\mathbf{\Psi}^f$ is an $NJ \times N$ matrix whose (nj, m)th element is $\frac{\pi^f_{nmj}P_m\mathcal{F}_m}{P_{nj}Y_{nj}}$. That is, this matrix stores the share of total revenue in the country-sector in the row that comes from final spending in the country in the column.
2. $\mathbf{\Psi}^x$ is an $NJ \times NJ$ matrix whose (nj, mi)th element is $\frac{(1-\eta_i)\pi^x_{nj,mi}P_{mi}Y_{mi}}{P_{nj}Y_{nj}}$. That is, this matrix stores the share of total revenue in the country-sector in the row that comes from intermediate spending in the country-sector in the column.
3. $\mathbf{\Upsilon}$ is an $N \times NJ$ matrix whose (n, mi)th element is $\frac{\eta_i P_{mi}Y_{mi}}{P_n\mathcal{F}_n}$. That is, this matrix stores the share of value added in the country-sector in the column in total GDP of the country in the row. Note that the elements are zero whenever $m \neq n$.
4. $\mathbf{\Pi}^f$ is an $NJ \times NJ$ matrix whose $(mi, k\ell)$th element is $\pi^f_{km\ell}$. That is, this matrix stores the final expenditure share in the country in the row from the goods coming from the country-sector in the column. The elements for different sectors in the row within the same country country are identical.
5. $\mathbf{\Pi}^c$ is an $N \times NJ$ matrix whose $(m, k\ell)$th element is $\pi^c_{km\ell}$. That is, this matrix stores the final expenditure share in the country in the row from the goods coming from the country-sector in the column. This share is within the goods in the sector in the column.

6. $\mathbf{\Pi}^x$ is an $NJ \times NJ$ matrix whose $(k\ell, mi)$th element is $\pi^x_{mi,k\ell}$. That is, this matrix stores the intermediate expenditure share on goods coming from the column in the country-sector in the row.

Then, equation (B.1) can be stated in matrix form:

$$\ln \mathbf{P}_t + \ln \mathbf{Y}_t = \left(\mathbf{\Psi}^f \mathbf{\Upsilon} + \mathbf{\Psi}^x\right)(\ln \mathbf{P}_t + \ln \mathbf{Y}_t) + (1-\rho)\left(\operatorname{diag}\left(\mathbf{\Psi}^f \mathbf{1}\right) - \mathbf{\Psi}^f \mathbf{\Pi}^c\right) \ln \mathbf{P}_t$$
$$+ (1-\varepsilon)\left(\operatorname{diag}\left(\mathbf{\Psi}^x \mathbf{1}\right) - \mathbf{\Psi}^x \mathbf{\Pi}^x\right) \ln \mathbf{P}_t.$$

This allows us to express prices as a function of quantities

$$\ln P_t = \mathcal{P} \ln Y_t, \tag{B.2}$$

where[36]

$$\mathcal{P} = -\left(\mathbf{I} - \mathcal{M}\right)^+ \left(\mathbf{I} - \mathbf{\Psi}^f \mathbf{\Upsilon} - \mathbf{\Psi}^x\right)$$
$$\mathcal{M} = \mathbf{\Psi}^f \mathbf{\Upsilon} + \mathbf{\Psi}^x + (1-\rho)\left(\operatorname{diag}\left(\mathbf{\Psi}^f \mathbf{1}\right) - \mathbf{\Psi}^f \mathbf{\Pi}^f\right) + (1-\varepsilon)\left(\operatorname{diag}\left(\mathbf{\Psi}^x \mathbf{1}\right) - \mathbf{\Psi}^x \mathbf{\Pi}^x\right).$$

Turning to the supply side, the labor demand (3.8) stacked into an $NJ \times 1$ vector is:

$$\ln \mathbf{W}_t - \ln \mathbf{P}_t = \ln \mathbf{Y}_t - \ln \mathbf{H}_t.$$

The log-linearized version of the final goods price in country n is

$$\ln P_{n,t} = \sum_j \nu_{nj} \sum_m \pi^c_{mnj} \ln P_{mj,t} = \sum_{m,j} \pi^f_{mnj} \ln P_{mj,t}.$$

The labor supply in vector notation is:

$$\ln \mathbf{H}_t = \psi(\ln \mathbf{W}_t - \ln \mathbf{P}^f_t),$$

where $\ln \mathbf{P}^f_t$ denotes the consumption price index (3.4) that prevails at each sector in log-deviations and matrix notation:

$$\ln \mathbf{P}^f_t = \mathbf{\Pi}^f \ln \mathbf{P}_t. \tag{B.3}$$

These three conditions imply the following equilibrium relationship for hours:

$$\ln \mathbf{H}_t = \frac{\psi}{1+\psi} \ln \mathbf{Y}_t + \frac{\psi}{1+\psi}\left(\mathbf{I} - \mathbf{\Pi}^f\right) \ln \mathbf{P}_t. \tag{B.4}$$

Similarly, market clearing for intermediate inputs is:

$$\ln \mathbf{P}^x_t - \ln \mathbf{P}_t = \ln \mathbf{Y}_t - \ln \mathbf{X}_t,$$

where $\ln \mathbf{P}^x_t$ is the vector of intermediate input price indices for all countries and sectors:

$$\ln \mathbf{P}^x_t = \mathbf{\Pi}^x \ln \mathbf{P}_t.$$

Jointly, these imply

$$\ln \mathbf{X}_t = \ln \mathbf{Y}_t + (\mathbf{I} - \mathbf{\Pi}^x) \ln \mathbf{P}_t.$$

[36]The + sign stands for the Moore-Penrose inverse as $\mathbf{I} - \mathcal{M}$ is not invertible. The non-invertibility is a consequence of the fact that the vector of prices is only defined up to a numeraire.

Plugging these into the production function

$$\begin{aligned}\ln \mathbf{Y}_t &= \ln \mathbf{Z}_t + \boldsymbol{\eta}\boldsymbol{\alpha} \ln \mathbf{H}_t + \boldsymbol{\eta}(\mathbf{I}-\boldsymbol{\alpha}) \ln \mathbf{K}_t + (\mathbf{I}-\boldsymbol{\eta}) \ln \mathbf{X}_t . \\ &= \ln \mathbf{Z}_t + \boldsymbol{\eta}(\mathbf{I}-\boldsymbol{\alpha}) \ln \mathbf{K}_t + \boldsymbol{\eta}\boldsymbol{\alpha} \left(\frac{\psi}{1+\psi} \ln \mathbf{Y} + \frac{\psi}{1+\psi} \left(\mathbf{I} - \boldsymbol{\Pi}^f\right) \ln \mathbf{P}_t \right) + (\mathbf{I}-\boldsymbol{\eta}) \left(\ln \mathbf{Y}_t + (\mathbf{I} - \boldsymbol{\Pi}^x) \ln \mathbf{P}_t\right). \\ &= \ln \mathbf{Z}_t + \boldsymbol{\eta}(\mathbf{I}-\boldsymbol{\alpha}) \ln \mathbf{K}_t + \left[\frac{\psi}{1+\psi} \boldsymbol{\eta}\boldsymbol{\alpha} \left(\mathbf{I} + \left(\mathbf{I}-\boldsymbol{\Pi}^f\right)\mathcal{P}\right) + (\mathbf{I}-\boldsymbol{\eta})\left(\mathbf{I} + (\mathbf{I}-\boldsymbol{\Pi}^x)\mathcal{P}\right)\right] \ln \mathbf{Y}_t ,\end{aligned}$$

where in the last step we use (3.16) for $\ln \mathbf{P}_t$. Inverting for $\ln \mathbf{Y}_t$ completes the proof:

$$\ln \mathbf{Y}_t = \left(\mathbf{I} - \frac{\psi}{1+\psi} \boldsymbol{\eta}\boldsymbol{\alpha} \left(\mathbf{I} + \left(\mathbf{I}-\boldsymbol{\Pi}^f\right)\mathcal{P}\right) - (\mathbf{I}-\boldsymbol{\eta})\left(\mathbf{I} + (\mathbf{I}-\boldsymbol{\Pi}^x)\mathcal{P}\right) \right)^{-1} \left(\ln \mathbf{Z}_t + \boldsymbol{\eta}(\mathbf{I}-\boldsymbol{\alpha}) \ln \mathbf{K}_t\right). \tag{B.5}$$

Proof of Proposition 3.2:. The log-deviation of country n's real GDP from steady state can be expressed as

$$\begin{aligned}\ln G_{n,t} &= \sum_j \left(\frac{P_{nj}Y_{nj}}{G_n} \ln Y_{nj,t} - \frac{P^x_{nj}X_{nj}}{G_n} \ln X_{nj,t} \right) \\ &= \sum_j \frac{P_{nj}Y_{nj}}{G_n} \left(\ln Y_{nj,t} - \frac{P^x_{nj}X_{nj}}{P_{nj}Y_{nj}} \ln X_{nj,t} \right) \\ &= \sum_j \frac{P_{nj}Y_{nj}}{G_n} \left(\ln Z_{nj,t} + \eta_j \alpha_j \ln H_{nj,t} + \eta_j(1-\alpha_j) \ln K_{nj,t} + (1-\eta_j) \ln X_{nj,t} - \frac{P^x_{nj}X_{nj}}{P_{nj}Y_{nj}} \ln X_{nj,t} \right),\end{aligned}$$

which leads directly to (3.17), since in equilibrium $\frac{P^x_{nj}X_{nj}}{P_{nj}Y_{nj}} = (1-\eta_j)$.

The derivation of (3.18) plugs (3.16) and (3.13)-(3.14) into (B.4), which leads to

$$\ln \mathbf{H}_t \equiv \boldsymbol{\mathcal{H}} \left(\ln \mathbf{Z}_t + (\boldsymbol{\eta} - \boldsymbol{\alpha}\boldsymbol{\eta}) \ln \mathbf{K}_t\right), \quad \mathcal{H} = \frac{\psi}{1+\psi} \left(\mathbf{I} + \left(\mathbf{I} - \boldsymbol{\Pi}^f\right) \boldsymbol{\mathcal{P}} \right) \boldsymbol{\Lambda}. \tag{B.6}$$

Proof of Corollary 3.3:. With $\rho = \varepsilon = 1$, the market clearing condition reduces to

$$\ln \mathbf{P}_t + \ln \mathbf{Y}_t = \left(\boldsymbol{\Psi}^f \boldsymbol{\Upsilon} + \boldsymbol{\Psi}^x \right) (\ln \mathbf{P}_t + \ln \mathbf{Y}_t),$$

which implies that

$$\ln \mathbf{P}_t = -\ln \mathbf{Y}_t, \qquad \mathcal{P} = -\mathbf{I}.$$

As a result, the influence matrix simplifies to

$$\ln \mathbf{Y}_t = \left(\mathbf{I} - \frac{\psi}{1+\psi} \boldsymbol{\eta}\boldsymbol{\alpha}\boldsymbol{\Pi}^f - (\mathbf{I}-\boldsymbol{\eta})\boldsymbol{\Pi}^x \right)^{-1} \left(\ln \mathbf{Z}_t + \boldsymbol{\eta}(\mathbf{I}-\boldsymbol{\alpha}) \ln \mathbf{K}_t\right). \tag{B.7}$$

The hours influence matrix is:

$$\mathcal{H} = \frac{\psi}{1+\psi} \left(\mathbf{I} + \left(\mathbf{I} - \boldsymbol{\Pi}^f\right) \boldsymbol{\mathcal{P}} \right) \boldsymbol{\Lambda} = \frac{\psi}{1+\psi} \boldsymbol{\Pi}^f \left(\mathbf{I} - \frac{\psi}{1+\psi} \boldsymbol{\eta}\boldsymbol{\alpha}\boldsymbol{\Pi}^f - (\mathbf{I}-\boldsymbol{\eta})\boldsymbol{\Pi}^x \right)^{-1}.$$

By construction, each row of $\boldsymbol{\Pi}^f$ and $\boldsymbol{\Pi}^x$ sum to one. Therefore, the largest eigenvalue of $\boldsymbol{\Pi}^f$ and $\boldsymbol{\Pi}^x$ equals to 1. Also note that both $\boldsymbol{\alpha}$ and $\boldsymbol{\eta}$ are diagonal matrix with elements less than 1. It follows that the largest eigenvalue of $\frac{\psi}{1+\psi}\boldsymbol{\eta}\boldsymbol{\alpha}\boldsymbol{\Pi}^f + (\mathbf{I}-\boldsymbol{\eta})\boldsymbol{\Pi}^x$ is bounded from above by $\frac{\psi}{1+\psi}$ which is less than 1. It follows that $\mathcal{H}$ admits the following expansion

$$\mathcal{H} = \frac{\psi}{1+\psi}\mathbf{\Pi}^f \sum_{k=0}^{\infty} \left(\frac{\psi}{1+\psi} \boldsymbol{\eta}\boldsymbol{\alpha}\mathbf{\Pi}^f + (\mathbf{I} - \boldsymbol{\eta})\mathbf{\Pi}^x \right)^k .$$

Proof of Proposition 3.4. In steady state, the intertemporal Euler condition can be written as

$$1 = \beta \left(\eta_j \alpha_j \frac{P_{nj} Y_{nj}}{P_n K_{nj}} + 1 - \delta_j \right).$$

Meanwhile, the budget constraint in the steady state is

$$P_n C_n + P_n \sum_j \delta_j K_{nj} = \sum_j \eta_j P_{nj} Y_{nj}.$$

These conditions imply that the steady state investment-to-output and consumption-to-output ratios can be expressed as

$$\begin{aligned} \frac{\delta_j K_{nj}}{G_n} &= \eta_j \alpha_j \omega_{nj} \frac{\delta_j}{\beta^{-1} - 1 + \delta_j}, \\ \frac{C_n}{G_n} &= \sum_j \eta_j \omega_{nj} \left(1 - \frac{\alpha_j \delta_j}{\beta^{-1} - 1 + \delta_j} \right), \end{aligned}$$

where G_n is GDP in country n and ω_{nj} is the Domar weight.

The steady state intratemporal Euler condition is

$$H_{nj}^{1+\frac{1}{\psi}} = \eta_j (1 - \alpha_j) \frac{P_{nj} Y_{nj}}{P_n},$$

which implies that

$$\frac{1}{G_n} \frac{H_{nj}^{1+\frac{1}{\psi}}}{1 + \frac{1}{\psi}} = \frac{\psi}{1+\psi} \eta_j (1 - \alpha_j) \omega_{nj}.$$

Now turn to the linearized version of the model. Starting from the budget constraint, we have

$$\frac{C_n}{G_n} \ln C_{n,t} + \sum_j \frac{\delta_j K_{nj}}{G_n} \ln I_{nj,t} = \sum_j \eta_j \omega_{nj} (\ln P_{nj,t} - \ln P_{n,t} + \ln Y_{nj,t})$$

Using the law of motion for capital accumulation, the changes in investment are given by

$$\ln I_{nj,t} = \frac{\ln K_{nj,t+1} - (1 - \delta_j) \ln K_{nj,t}}{\delta_j}.$$

Therefore, the change of consumption can be written in terms of $\ln \mathbf{Y}_t$, $\ln K_{nj,t}$, and $\ln K_{nj,t+1}$, as changes in prices can be expressed as a function of $\ln \mathbf{Y}_t$

$$\frac{C_n}{G_n} \ln C_{n,t} = \sum_j \eta_j \omega_{nj} (\ln P_{nj,t} - \ln P_{n,t} + \ln Y_{nj,t}) - \sum_j \frac{\delta_j K_{nj}}{G_n} \left(\frac{\ln K_{nj,t+1} - (1 - \delta_j) \ln K_{nj,t}}{\delta_j} \right). \tag{B.8}$$

Let ϕ_{nt} denote the deviation of the marginal utility of consumption

$$\phi_{n,t} = \left(C_{n,t} - \sum_j \frac{H_{nj,t}^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}} \right)^{-\sigma}$$

Utilizing the steady-state properties of consumption and hours, it follows that its linearized version is

$$\ln \phi_{n,t} \equiv -\sigma \left(\frac{C_n \ln C_{n,t} - \sum_j \frac{H_{nj}^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}} \left(1+\frac{1}{\psi}\right) \ln H_{nj,t}}{C_n - \sum_j \frac{H_{nj}^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}}} \right) = -\sigma \frac{\sum_j \eta_j \omega_{nj} \left(\left(1 - \frac{\alpha_j \delta_j}{\beta^{-1}-1+\delta_j}\right) \ln C_{n,t} - (1-\alpha_j) \ln H_{nj,t} \right)}{\sum_j \eta_j \omega_{nj} \left(1 - \frac{\alpha_j \delta_j}{\beta^{-1}-1+\delta_j} - \frac{\psi}{1+\psi}(1-\alpha_j)\right)}$$

The linearized Euler equation is

$$\ln \phi_{n,t} = \mathbb{E}_t \left[\phi_{n,t+1} + (1-\beta+\beta\delta_j)(\ln P_{nj,t+1} - \ln P_{n,t+1} + \ln Y_{nj,t+1} - \ln K_{nj,t+1}) \right]. \tag{B.9}$$

Combining condition (B.2), (B.3), (B.5), (B.6), and (B.8) with condition (B.9), it boils down to a second-order difference system involving only $\ln \mathbf{K}_t$, $\ln \mathbf{K}_{t+1}$, $\ln \mathbf{K}_{t+2}$, $\ln \mathbf{Z}_t$, and $\ln \mathbf{Z}_{t+1}$

$$\mathbf{D}_0 \ln \mathbf{K}_t + \mathbf{D}_1 \ln \mathbf{K}_{t+1} + \mathbf{D}_2 \mathbb{E}[\ln \mathbf{K}_{t+2}] + \mathbf{F}_1 \ln \mathbf{Z}_t + \mathbf{F}_2 \mathbb{E}[\ln \mathbf{Z}_{t+1}] = \mathbf{0}, \tag{B.10}$$

where the matrices $\mathbf{D}_0, \mathbf{D}_1, \mathbf{D}_2, \mathbf{F}_1, \mathbf{F}_2$ are functions of the parameters and the steady-state input and output shares.

We proceed by the method of undetermined coefficients. Suppose that

$$\ln \mathbf{K}_{t+1} = \mathbf{M} \ln \mathbf{K}_t + \boldsymbol{\Gamma} \ln \mathbf{Z}_t. \tag{B.11}$$

We assume that the TFP shock obeys the following autoregressive process,

$$\ln \mathbf{Z}_t = \boldsymbol{\varrho} \ln \mathbf{Z}_{t-1} + \boldsymbol{\theta}_t, \tag{B.12}$$

where $\boldsymbol{\varrho}$ governs the persistence of the system and $\boldsymbol{\theta}_t$ are a vector of innovations that are serially uncorrelated.

Plugging equations (B.11) and (B.12) into (B.10) leads to

$$\mathbf{D}_0 \ln \mathbf{K}_t + \mathbf{D}_1(\mathbf{M} \ln \mathbf{K}_t + \boldsymbol{\Gamma} \ln \mathbf{Z}_t) + \mathbf{D}_2(\mathbf{M}(\mathbf{M} \ln \mathbf{K}_t + \boldsymbol{\Gamma} \ln \mathbf{Z}_t) + \boldsymbol{\Gamma} \boldsymbol{\varrho} \ln \mathbf{Z}_t) + \mathbf{F}_1 \ln \mathbf{Z}_t + \mathbf{F}_2 \boldsymbol{\varrho} \ln \mathbf{Z}_t = \mathbf{0}.$$

This condition above needs to hold for all possible realization of $\ln \mathbf{K}_t$ and $\ln \mathbf{Z}_t$. The resulting fixed-point problem is

$$\mathbf{D}_0 + \mathbf{D}_1 \mathbf{M} + \mathbf{D}_2 \mathbf{M}^2 = 0 \tag{B.13}$$
$$(\mathbf{D}_1 + \mathbf{D}_2 \mathbf{M}) \boldsymbol{\Gamma} + \mathbf{D}_2 \boldsymbol{\Gamma} \boldsymbol{\varrho} + \mathbf{F}_1 + \mathbf{F}_2 \boldsymbol{\varrho} = 0. \tag{B.14}$$

Note that $\mathbf{M}$ can be solved separately using only condition (B.13). The solution to this second-order equation in matrix form needs to satisfy the condition that all the eigenvalues of $\mathbf{M}$ have to be less than 1 in norm. Once $\mathbf{M}$ is obtained, $\boldsymbol{\Gamma}$ can be solved using condition (B.14).

Rearranging the policy function (B.11) gives

$$\ln \mathbf{K}_{t+1} = \mathbf{M} \mathbb{L} \ln \mathbf{K}_t + \boldsymbol{\Gamma} \ln \mathbf{Z}_t = (\mathbf{I} - \mathbf{M}\mathbb{L})^{-1} \boldsymbol{\Gamma} \ln \mathbf{Z}_t.$$

Proof of Proposition 3.5:. Under full depreciation, we use a guess-and-verify approach to show that the saving rate is constant in each country. Suppose the consumption rate in country n is $1-\chi$, then

$$C_{n,t} = \frac{1}{P_{n,t}}(1-\chi) \sum_j \eta_j P_{nj,t} Y_{nj,t}.$$

Meanwhile, the intratemporal Euler equation implies that

$$H_{nj,t}^{\frac{1}{\psi}} = \frac{1}{P_{n,t}} \frac{\psi}{1+\psi} \alpha \eta_j P_{nj,t} Y_{nj,t},$$

where we have invoked the assumption that $\alpha_j = \alpha$. When $U(\cdot) = \ln(\cdot)$, that the marginal utility of consumption is

$$\phi_{n,t} = \frac{1}{C_{n,t} - \sum_j H_{nj,t}^{1+\frac{1}{\psi}}} = \frac{P_{n,t}}{(1-\chi)\sum_j \eta_j P_{nj,t} Y_{nj,t} - \frac{\psi}{1+\psi}\alpha \sum_j \eta_j P_{nj,t} Y_{nj,t}}$$

The Euler condition requires that

$$\phi_{n,t} = \beta \mathbb{E}_t \left[\phi_{n,t+1} \left(\frac{R_{nj,t+1}}{P_{n,t+1}} \right) \right].$$

Using the equilibrium condition that $K_{nj,t} R_{nj,t} = (1-\alpha)\eta_j P_{nj,t} Y_{nj,t}$, the Euler equation becomes

$$\frac{P_{n,t} K_{nj,t+1}}{(1-\chi)\sum_j \eta_j P_{nj,t} Y_{nj,t} - \frac{\psi}{1+\psi}\sum_j \alpha \eta_j P_{nj,t} Y_{nj,t}} = \beta \mathbb{E} \left[\frac{(1-\alpha)\eta_j P_{nj,t+1} Y_{nj,t+1}}{(1-\chi)\sum_j \eta_j P_{nj+1t} Y_{nj,t+1} - \frac{\psi}{1+\psi}\alpha \sum_j \eta_j P_{nj,t+1} Y_{nj,t+1}} \right].$$

Guess that each sector's investment is a fixed fraction of total GDP

$$P_{n,t} K_{nj,t+1} = \chi_j \sum_j \eta_j P_{nj,t} Y_{nj,t},$$

where $\sum_j \chi_j = \chi$. The Euler equation can be simplified as

$$\frac{\chi_j}{1-\chi-\alpha\frac{\psi}{1+\psi}} = \beta \frac{(1-\alpha)\eta_j}{1-\chi-\alpha\frac{\psi}{1+\psi}},$$

which leads to

$$\chi_j = \beta(1-\alpha)\eta_j, \quad \text{and} \quad \chi = \beta(1-\alpha)\sum_j \eta_j.$$

As a result, the changes in capital $K_{nj,t}$ is proportional to the changes in GDP $G_{n,t}$

$$\ln K_{nj,t+1} = \ln G_{n,t}.$$

The dynamics of GDP can therefore be represented as

$$\begin{aligned}
G_{n,t} &= \sum_j \omega_{nj} \left(\ln Z_{nj,t} + \alpha_j \eta_j \ln H_{nj,t} + (1-\alpha_j)\eta_j \ln K_{nj,t} \right) \\
&= \sum_j \omega_{nj} \left(\ln Z_{nj,t} + \alpha_j \eta_j \ln H_{nj,t} + (1-\alpha_j)\eta_j \ln G_{n,t-1} \right) \\
&= \sum_j \omega_{nj} \left(\ln Z_{nj,t} + (1-\alpha_j)\eta_j \ln G_{n,t-1} + \alpha_j \eta_j \sum_{m,i} \mathcal{H}_{nj,mi} \left(\ln Z_{mi,t} + \eta_i (1-\alpha_i) \ln K_{mi,t} \right) \right) \\
&= \sum_j \omega_{nj} \left(\ln Z_{nj,t} + (1-\alpha_j)\eta_j \ln G_{n,t-1} + \alpha_j \eta_j \sum_{m,i} \mathcal{H}_{nj,mi} \left(\ln Z_{mi,t} + \eta_i (1-\alpha_i) \ln G_{m,t-1} \right) \right).
\end{aligned}$$

Proof of Proposition 5.1:. First consider the real wage in the baseline economy without wage rigidity. The labor supply condition is

$$\ln \mathbf{H}_t = \psi \ln \boldsymbol{w}_t^*.$$

This implies that

$$\ln \boldsymbol{w}_t^* = \frac{1}{\psi}\mathcal{H}(\ln \mathbf{Z}_t + (\mathbf{I}-\boldsymbol{\eta})\boldsymbol{\alpha}\ln \mathbf{K}_t) = \frac{1}{1+\psi}\left(\mathbf{I}+\left(\mathbf{I}-\boldsymbol{\Pi}^f\right)\boldsymbol{\mathcal{P}}\right)\boldsymbol{\Lambda}(\psi),$$

where

$$\boldsymbol{\Lambda}(x) = \left(\mathbf{I} - \frac{x}{1+x}\boldsymbol{\eta}\boldsymbol{\alpha}\left(\mathbf{I}+\left(\mathbf{I}-\boldsymbol{\Pi}^f\right)\boldsymbol{\mathcal{P}}\right) - (\mathbf{I}-\boldsymbol{\eta})\left(\mathbf{I}+(\mathbf{I}-\boldsymbol{\Pi}^x)\boldsymbol{\mathcal{P}}\right)\right)^{-1}.$$

Particularly,

$$\boldsymbol{\Lambda}(\infty) = \left(\mathbf{I} - \boldsymbol{\eta}\boldsymbol{\alpha}\left(\mathbf{I}+\left(\mathbf{I}-\boldsymbol{\Pi}^f\right)\boldsymbol{\mathcal{P}}\right) - (\mathbf{I}-\boldsymbol{\eta})\left(\mathbf{I}+(\mathbf{I}-\boldsymbol{\Pi}^x)\boldsymbol{\mathcal{P}}\right)\right)^{-1}.$$

Now consider wage rigidity. The real wage obeys

$$\ln \boldsymbol{w}_t = (1-\lambda)\ln \boldsymbol{w}_t^* + \lambda \ln \boldsymbol{w}_{t-1}. \tag{B.15}$$

The labor market is demand determined, which leads to

$$\ln \boldsymbol{w}_t + \ln \mathbf{P}_t^f - \ln \mathbf{P}_t = \ln \mathbf{Y}_t - \ln \mathbf{H}_t. \tag{B.16}$$

Meanwhile, combining the production function and the optimal demand for intermediate goods gives

$$\ln \mathbf{Y}_t = \ln \mathbf{Z}_t + \boldsymbol{\eta}\boldsymbol{\alpha}\ln \mathbf{H}_t + \boldsymbol{\eta}(\mathbf{I}-\boldsymbol{\alpha})\ln \mathbf{K}_t + (\mathbf{I}-\boldsymbol{\eta})\left(\ln \mathbf{Y}_t + (\mathbf{I}-\boldsymbol{\Pi}^x)\ln \mathbf{P}_t\right). \tag{B.17}$$

When substituting condition (B.15) and (B.16) into condition (B.17), we have

$$\ln \mathbf{Y}_t = \mathbf{A}(\ln \mathbf{Z}_t + \boldsymbol{\eta}(\mathbf{I}-\boldsymbol{\alpha})\ln \mathbf{K}_t) - \lambda\boldsymbol{\Lambda}(\infty)\boldsymbol{\alpha}\boldsymbol{\eta}\ln \boldsymbol{w}_{t-1},$$

where $\mathbf{A}$ denotes the impact response

$$\mathbf{A} \equiv \boldsymbol{\Lambda}(\infty)\left(\mathbf{I} - \frac{1-\lambda}{1+\psi}\boldsymbol{\alpha}\boldsymbol{\eta}\left(\mathbf{I}+\left(\mathbf{I}-\boldsymbol{\Pi}^f\right)\boldsymbol{\mathcal{P}}\right)\boldsymbol{\Lambda}(\psi)\right).$$

By the definition of the matrix function $\boldsymbol{\Lambda}(x)$, the impact response satisfies

$$\begin{aligned}
\boldsymbol{\Lambda}(\infty)^{-1}\mathbf{A}\boldsymbol{\Lambda}(\psi)^{-1} &= \boldsymbol{\Lambda}(\psi)^{-1} - \frac{1-\lambda}{1+\psi}\boldsymbol{\alpha}\boldsymbol{\eta}\left(\mathbf{I}+\left(\mathbf{I}-\boldsymbol{\Pi}^f\right)\boldsymbol{\mathcal{P}}\right)\\
&= \mathbf{I} - \frac{\psi+1-\lambda}{1+\psi}\boldsymbol{\eta}\boldsymbol{\alpha}\left(\mathbf{I}+\left(\mathbf{I}-\boldsymbol{\Pi}^f\right)\boldsymbol{\mathcal{P}}\right) - (\mathbf{I}-\boldsymbol{\eta})\left(\mathbf{I}+(\mathbf{I}-\boldsymbol{\Pi}^x)\boldsymbol{\mathcal{P}}\right)\\
&= \boldsymbol{\Lambda}\left(\frac{\psi+1-\lambda}{\lambda}\right)^{-1}.
\end{aligned}$$

Therefore, we have the desired result

$$\mathbf{A} = \boldsymbol{\Lambda}(\infty)\boldsymbol{\Lambda}\left(\frac{\psi+1-\lambda}{\lambda}\right)^{-1}\boldsymbol{\Lambda}(\psi).$$

Special case with complete markets. With complete markets, the relative marginal utility of consumption equals the real exchange rate

$$\frac{C_{n,t}^{-\sigma}}{C_{m,t}^{-\sigma}} = \frac{P_{n,t}}{P_{m,t}},$$

which implies

$$\ln P_{n,t} + \sigma \ln C_{n,t} = \ln P_{m,t} + \sigma \ln C_{m,t}.$$

Denote χ_t as

$$\chi_t \equiv \ln P_{n,t} + \sigma \ln C_{n,t}.$$

When normalizing the world nominal GDP to be 1, it follows that

$$\sum_m P_{m,t} C_{m,t} = 1,$$

which implies that

$$\sum_m P_m C_m (\ln P_{m,t} + \ln C_{m,t}) = 0.$$

When $\sigma = 1$, this condition reduces to

$$\chi_t = 0.$$

Also note that under the elasticities of substitution equal to 1 ($\rho = \varepsilon = 1$), the expenditure shares for both final goods and intermediate goods remain constant, which implies that for any country n sector j

$$\ln P_{nj,t} + \ln Y_{nj,t} = 0.$$

In labor markets, the labor supply satisfies the intratemporal Euler condition

$$C_{n,t}^{\sigma} H_{nj,t}^{\frac{1}{\psi}} = \frac{W_{nj,t}}{P_{n,t}},$$

and the linearized version is

$$\ln P_{n,t} + \sigma \ln C_{n,t} + \frac{1}{\psi} \ln H_{nj,t} = \ln W_{nj,t}.$$

When $\sigma = 1$, it follows that $\ln P_{n,t} + \ln C_{n,t} = 0$ and

$$\frac{1}{\psi} \ln H_{nj,t} = \ln W_{nj,t}.$$

The labor demand condition requires

$$\ln W_{nj,t} + H_{nj,t} = \ln P_{nj,t} + \ln Y_{nj,t}.$$

Since $\ln P_{nj,t} + \ln Y_{nj,t} = 0$ when $\rho = \varepsilon = 1$, it follows that

$$\ln H_{nj,t} = \ln W_{nj,t} = 0.$$

That is, hours remain constant in response to shocks, and the international comovements are only driven by correlated shocks.

B.2 Dynamic Model Solution Details

In this subsection, we provide more detail on the numerical algorithm in computing the dynamic model. An important feature of our problem is that the stochastic processes of the shocks are not known ex ante and need to be estimated based on the realized shocks, while the realized shocks are the result of the model inversion which requires the shock processes in the first place. That is, the shock processes are the solution to a fixed-point problem.

To make sure that the perceived laws of motion of both the shocks and endogenous equilibrium outcomes are consistent with the actual laws of motion, we apply the following iterative algorithm:

1. Solve for the matrix $\mathbf{M}$ using condition (B.13), which does not require the shock processes.
   (a) Make an initial guess $\mathbf{M}^{(0)}$.
   (b) Update the guess according to
$$\mathbf{M}^{(1)} = -\mathbf{D}_1^{-1}\left(\mathbf{D}_0 + \mathbf{D}_2\left(\mathbf{M}^{(0)}\right)^2\right)$$
   (c) Replace the initial guess by $\mathbf{M}^{(1)}$ and iterate until convergence.
2. Solve for the matrix $\mathbf{\Gamma}$ and estimate the shock process $\boldsymbol{\varrho}$ jointly.
   (a) Make an initial guess $\boldsymbol{\varrho}^{(0)}$ that governs the autoregressive model (B.12).
   (b) Solve for the matrix $\mathbf{\Gamma}$ that satisfies condition (B.14):
      i. Make an initial guess $\mathbf{\Gamma}^{(0)}$
      ii. Update the guess according to
$$\mathbf{\Gamma}^{(1)} = -(\mathbf{D}_1 + \mathbf{D}_2\mathbf{M})^{-1}\left(\mathbf{D}_2\mathbf{\Gamma}^{(0)}\boldsymbol{\varrho}^{(0)} + \mathbf{F}_1 + \mathbf{F}_2\boldsymbol{\varrho}^{(0)}\right)$$
      iii. Replace the initial guess by $\mathbf{\Gamma}^{(1)}$ and iterate until $\mathbf{\Gamma}^{(0)}$ and $\mathbf{\Gamma}^{(1)}$ converge.
   (c) Obtain the shocks that match the observed value added growth at each country sector. Recall that by Proposition 3.2, the vector of value added is given by
$$\ln \mathbf{V}_t = \ln \mathbf{Z}_t + \boldsymbol{\alpha\eta} \ln \mathbf{H}_t + (\mathbf{I} - \boldsymbol{\alpha})\boldsymbol{\eta} \ln \mathbf{K}_t.$$
   Assume that the shocks before the first observation of the data are zero. It follows that
$$\ln \mathbf{V}_0 = \ln \mathbf{Z}_0 + \boldsymbol{\alpha}\eta\mathcal{H} \ln \mathbf{Z}_0,$$
   and for $t > 0$
$$\ln \mathbf{V}_t = \ln \mathbf{Z}_t + \boldsymbol{\alpha\eta}\mathcal{H}\left(\ln \mathbf{Z}_t + \sum_{k=0}^{t-1} \mathbf{M}^{t-k-1}\mathbf{\Gamma} \ln \mathbf{Z}_k\right) + (\mathbf{I} - \boldsymbol{\alpha})\boldsymbol{\eta}\left(\ln \mathbf{Z}_t + \sum_{k=0}^{t-1} \mathbf{M}^{t-k-1}\mathbf{\Gamma} \ln \mathbf{Z}_k\right),$$
   where the sequence of capital is given by
$$\ln \mathbf{K}_t = \sum_{k=0}^{t-1} \mathbf{M}^{t-k-1}\mathbf{\Gamma} \ln \mathbf{Z}_k.$$
   By matching the data on $\ln \mathbf{V}_t$, one can recover the sequence of shocks $\{\ln \mathbf{Z}_t\}$.
   (d) Estimate the new shock process according to
$$\ln Z_{nj,t} = \rho_{nj} \ln Z_{nj,t-1} + \zeta_n \mathbf{1}(m = n, k \neq j) \ln Z_{mk,t-1} + \theta_{nj,t},$$
   where we allow the dependence on its own past value and the past values within the same country. The estimated coefficients enter the new shock process matrix $\boldsymbol{\varrho}^{(1)}$.
   (e) Replace the initial guess of the shock process by $\boldsymbol{\varrho}^{(1)}$ and iterate until convergence.

This algorithm makes sure that the perceived laws of motion and the actual laws of motion are the same.

B.3 Exact Solution to the Static Network Model

This section sets up the exact solution to the static network model, in changes, following the methodology of Dekle, Eaton, and Kortum (2008). In this section, denote by a "hat" the gross proportional change in any variable between the steady state x and a counterfactual x_t: $\widehat{x} \equiv x_t/x$. To streamline notation, define $\mathcal{Y}_{nj,t} \equiv P_{nj,t}Y_{nj,t}$ to be the gross revenue in sector j, country n. Following a set of supply shocks $\widehat{Z}_{nj,t}$, the price in sector j, country n experiences the change:

$$\widehat{P}_{nj,t} = \widehat{Z}_{nj,t}^{-1}\widehat{\mathcal{Y}}_{nj,t}^{(1-\alpha_j)\eta_j - \frac{1}{\psi+1}\alpha_j\eta_j}\widehat{P}_{n,t}^{\alpha_j\eta_j\frac{\psi+1}{\psi}}\left(\sum_{m,i}\pi_{mi,nj}\widehat{P}_{mi,t}^{1-\varepsilon}\right)^{\frac{1-\eta_j}{1-\varepsilon}}. \tag{B.18}$$

This, together with the dependence of $\widehat{P}_{n,t}$ on the constituent $\widehat{P}_{ni,t}$:

$$\widehat{P}_{n,t} = \left[\sum_i\sum_m \widehat{P}_{mi,t}^{1-\rho}\pi_{mni}^f\right]^{\frac{1}{1-\rho}} \tag{B.19}$$

defines a system of $J \times N$ equations in prices, conditional on known steady-state data quantities (such as π_{mni}^f), and a vector of $\widehat{\mathcal{Y}}_{nj,t}$'s. The price changes in turn determine the counterfactual shares (denoted by a t-subscript):

$$\pi_{nmj,t}^f = \frac{\widehat{P}_{nj,t}^{1-\rho}\pi_{nmj}^f}{\sum_k \widehat{P}_{kj,t}^{1-\rho}\pi_{kmj}^f}, \tag{B.20}$$

$$\pi_{nj,mi,t}^x = \frac{\widehat{P}_{nj,t}^{1-\varepsilon}\pi_{nj,mi}^x}{\sum_{k,\ell}\widehat{P}_{k\ell,t}^{1-\varepsilon}\pi_{k\ell,mi}^x}. \tag{B.21}$$

These trade shares have to be consistent with market clearing at the counterfactual t, expressed using proportional changes as:

$$\widehat{\mathcal{Y}}_{nj,t}\mathcal{Y}_{nj} = \sum_m\left[\pi_{nmj,t}^f\omega_{jm}\left(\sum_i\eta_i\widehat{\mathcal{Y}}_{mi,t}\mathcal{Y}_{mi}\right) + \sum_i\pi_{nj,mi,t}^x(1-\eta_i)\widehat{\mathcal{Y}}_{mi,t}\mathcal{Y}_{mi}\right]. \tag{B.22}$$

The sets of equations (B.18)-(B.22) represent a system of $2 \times N \times J + N^2 \times J + N^2 \times J^2$ unknowns, $\widehat{P}_{nj,t}\ \forall n,j$, $\widehat{\mathcal{Y}}_{nj,t}\ \forall n,j$, $\pi_{nmj,t}^f\ \forall n,m,j$, and $\pi_{nj,mi,t}^x\ \forall n,j,m,i$ that is solved under given parameter values and under a set of shocks $\widehat{Z}_{nj,t}$.

B.3.1 Algorithm for Exact Solution to the Static Model

To solve the model, we use an initial guess for $\widehat{\mathcal{Y}}_{nj,t}$ together with data on π_{mnj}^f and $\pi_{nj,mi}^x$. Given these variables, the algorithm is as follows:

1. Solve for $\widehat{P}_{nj,t}$ given the guess of $\widehat{\mathcal{Y}}_{nj}$ and the data on π_{mnj}^f and $\pi_{nj,mi}^x$. This step uses equations (B.18) and (B.19) .
2. Update $\pi_{mnj,t}^f$ and $\pi_{nj,mi,t}^x$ given the solution to step 1 using equations (B.20) and (B.21).
3. Solve for the new guess $\widehat{\mathcal{Y}}'_{nj,t}$ using equation (B.22) given the prices $\widehat{P}_{nj,t}$ obtained in step 1 and the updated shares $\pi_{mnj,t}^f$ and $\pi_{nj,mi,t}^x$ from step 2.
4. Check if $\max|(\widehat{\mathcal{Y}}'_{nj,t} - \widehat{\mathcal{Y}}_{nj,t})| < \delta$, where δ is a tolerance parameter that is arbitrarily small. If not, update the guess of $\widehat{\mathcal{Y}}_{nj,t}$ and repeat steps 1-4 until convergence.

Table A3: First-Order and Exact Solutions: Correlations of $d\ln G_{n,t}$, G7 Countries

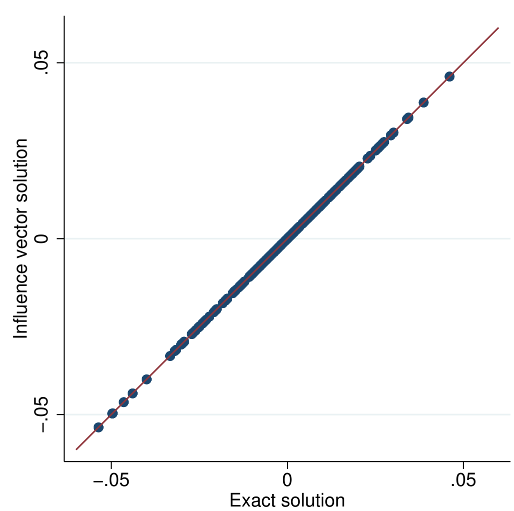
	Mean	Median	25th pctile	75th pctile
		N. obs. = 21		
Static model (approx.)	0.350	0.356	0.124	0.558
Exact solution	0.350	0.356	0.230	0.558

Notes: This table presents the summary statistics of the correlations of the model $d\ln G_{n,t}$ in the sample of G7 countries computed using the linear approximation and the exact solution. Variable definitions and sources are described in detail in the text.

B.3.2 Comparison of the Exact and First-Order Solutions

Figure A3 presents a scatterplot of GDP growth rates obtained under the first-order analytical solution to the global influence matrix in Section 3.2 against the exact solution computed as in this appendix. The line through the data is the 45-degree line. The GDP growth rates are computed under the observed shocks, and pooled across countries and years. It is clear that the first-order approximation is quite good in essentially every country-year instances. The correlation between the two sets of growth rates is 0.998. Table A3 summarizes the GDP correlations obtained using GDP growth rates in the linear and exact solutions. The correlations are very close to each other.

Figure A3: Comparison of GDP Growth Rates between First-Order and Exact Solutions

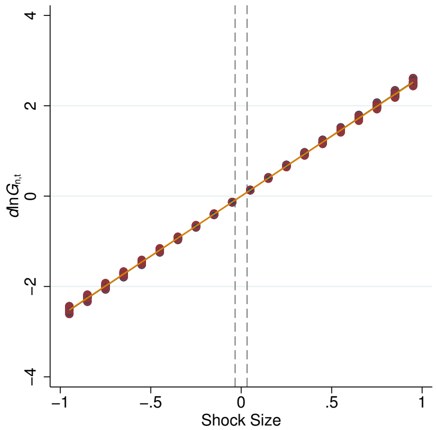


Notes: This figure displays a scatterplot of the GDP growth rates obtained using the first-order approximation against the GDP growth rates in the exact solution to the model, pooling countries and years. The line through the data is the 45-degree line.

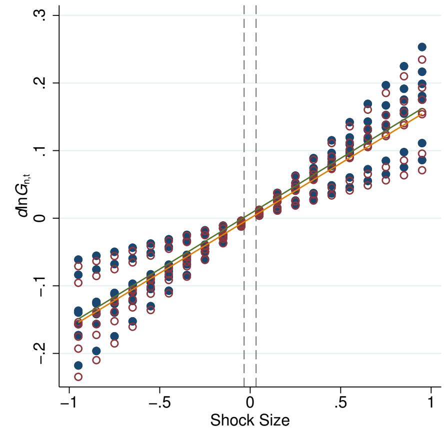
B.3.3 When is the First-Order Solution Reasonable?

We next turn to the question of when using the first-order solution provides a reasonable approximation to the exact solution. Baqaee and Farhi (2019a) argue that higher-order effects can be quantitatively relevant in non-linear network economies, particularly when the shock size is large. Figure A3 illustrated that for our

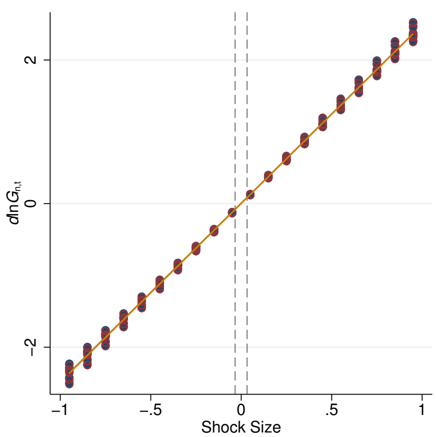
Figure A4: Comparison of GDP Growth Rates between First-Order and Exact Solutions for Varied Shock Sizes



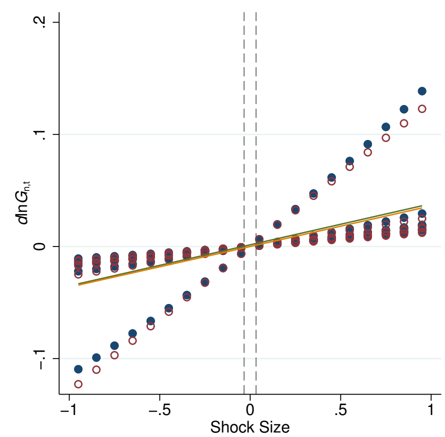
(a) Global Shock



(b) Rest-of-the-World Shock



(c) Domestic Shock



(d) US Shock

Notes: This figure displays the GDP growth rates obtained using the exact solution (solid blue dots) and the first-order approximation to the model (hollow red circles) for varied shock sizes under different scenarios. Panel (a) shows the results for all countries when every country and sector has the same sized shock. Panel (b) shows the results when every country and sector except the country in question has a same sized shock ("rest-of-the world shock"). Panel (c) shows the results when only the country in question has the same sized shock in all sectors ("domestic shock") and Panel (d) shows the results when only the US has a shock in every sector ("US Shock"). The solid lines show the cross-country averages at each size shock. The vertical dashed lines show the 5th and 95th percentiles of the actual shocks recovered from the G7 data over the period 1978-2007.

actual recovered shocks, the exact and first-order solution are nearly identical. To explore the size of shocks for which the linear solution would yield a worse approximation, Figure A4 plots the log changes in GDP under the first order (red hollow circles) and exact (solid blue dots) for all countries following 4 shocks. The lines plot the averages across countries for each size shock. In each case, the size of the shocks, in logs, is on the x-axis. The first is a "Global" shock, where all country-sectors receive a shock of the same size. This is an extreme example of a perfectly correlated shock. The second is a scenario in which the "Rest-of-the-World" receives a shock, but the domestic country in question is not shocked. The third is a purely domestic shock, while all trade partners receive no shock. Finally, we consider the responses of all countries to a US shock. In all exercises, the size of the shock is symmetric across sectors within a country that is shocked, and across countries in the instance where several countries are shocked. In the Rest-of-the-World and Domestic scenarios, we solve the exact model country by country. For reference, we also include lines illustrating the 5th and 95th percentile of the sectoral shocks recovered in the data.

It is immediate that non-linearities increase in importance for larger shock sizes. This is true for all four types of shocks. However, there are a number of interesting nuances. First, in the case of the Global shock, even for extremely large shocks (a 50-95% increase or decrease in $d \ln Z$), the difference in the exact and linearized solution is fairly small. Intuitively, in the case of a common shock, there is little relative change in sectoral sizes across countries, which decreases the size of the higher-order terms missed by the linear approximation. In the context of even large correlated shocks such as the Covid-19 pandemic, this suggests that our influence matrix analytical solution is useful. Second, the difference between the exact and linearized model can be large but only in when the shock sizes are very large (90-95%), and induce large changes in relative sector sizes, as is true in panels (b) and (d) when either the rest-of-the world is shocked or there is a very large US shock. In the case of a US shock, only its closest trading partner Canada sees a noticeable discrepancy between the exact and first-order solution, and that is still only about 5 percentage points for a US shock of 95% in all sectors.

This discussion suggests that while non-linearities can be quantitatively important in some extreme cases, in the empirically relevant range of shocks observed in the data using the linearized model is reasonable. Additionally, the shocks recovered in Section 4 were found to be very correlated. Our analysis illustrates that this is precisely the instance where, even if the shocks are larger, the linearized model is a good approximation of the exact solution.

C. Estimation

C.1 Estimation of ρ and ε

We use model-implied relationships to estimate ρ and ε. In this section, denote by a "hat" the gross proportional change in any variable between time t and the previous year: $\widehat{x}_t \equiv x_t/x_{t-1}$. Further assume that iceberg trade costs, final consumer taste shocks, and input share shocks have a stochastic element, and denote their gross proportional changes by $\widehat{\tau}_{mnj,t}$, $\widehat{\vartheta}_{mnj,t}$, and $\widehat{\mu}_{mj,ni,t}$, respectively. This helps interpret the error term in the estimating equations. Log-differencing the CES expenditure shares (3.5) and (3.10) first with respect to $t-1$ and then with respect to a reference country m' yields the following relationships between shares and prices:

$$\ln\left(\frac{\widehat{\pi}^f_{mnj,t}}{\widehat{\pi}^f_{m'nj,t}}\right) = (1-\rho)\ln\left(\frac{\widehat{P}_{mj,t}}{\widehat{P}_{m'j,t}}\right) + \ln\left(\frac{\widehat{\vartheta}_{mnj,t}\widehat{\tau}^{1-\rho}_{mnj,t}}{\widehat{\vartheta}_{m'nj,t}\widehat{\tau}^{1-\rho}_{m'nj,t}}\right) \tag{C.1}$$

and

$$\ln\left(\frac{\widehat{\pi}^x_{mj,ni,t}}{\widehat{\pi}^x_{m'j,ni,t}}\right) = (1-\varepsilon)\ln\left(\frac{\widehat{P}_{mj,t}}{\widehat{P}_{m'j,t}}\right) + \ln\left(\frac{\widehat{\mu}_{mj,ni,t}\widehat{\tau}^{1-\varepsilon}_{mnj,t}}{\widehat{\mu}_{m'j,ni,t}\widehat{\tau}^{1-\varepsilon}_{m'nj,t}}\right). \tag{C.2}$$

We express the expenditure share change $\widehat{\pi}^f_{mnj,t}$ relative to the expenditure share change in a reference country m'. In the baseline estimation, this reference country is chosen separately for each importing country-sector (n,j) as the country with the largest average expenditure share in that country-sector. (Thus, strictly speaking, the identity of the reference country m' is distinct for each importing country-sector, but we suppress the dependence of m' on (n,j) to streamline notation.)

The estimation amounts to regressing double-differenced expenditure share changes on relative price changes. A threat to identification would be that relative source country price changes are affected by destination country demand shocks (e.g. $\widehat{\vartheta}_{mnj,t}$), and thus correlated with the residual. We address endogeneity by means of three strategies. First, to absorb as much of the confounding variation as possible, we include source-destination-reference country-time ($n \times m \times m' \times t$) fixed effects. These absorb many possible confounders including any common components occurring at the country 3-tuple-time level, such as exchange rate changes and other taste and transport cost changes, and thus the coefficient is estimated from the variation across sectors in relative price indices and relative share movements within that cell.

Second, our estimates are based on the subsample in which destination countries are all non-G7, and the source and reference countries are all G7 countries. In this sample it is unlikely that taste shocks in the (smaller) destination countries will affect relative price changes in the larger G7 source countries. This sample restriction by construction also implies that all the own expenditure shares $\widehat{\pi}^f_{nnj,t}$ are dropped from the estimation sample. This is desirable since the domestic shares are computed as residuals in WIOD, whereas import shares from other countries are taken directly from the international trade data. To reduce the impact of small shares on the estimates, we report results weighting by the size of the beginning-of-period expenditure shares ($\pi^f_{mnj,1995}$ and $\pi^x_{mj,ni,1995}$), and consider alternative weights as robustness checks.

Third, in addition to the steps described above to mitigate endogeneity issues, we use TFP shocks as instruments for changes in relative prices. Thus, we use for estimation only the variation in the relative price change $\widehat{P}_{mj,t}/\widehat{P}_{m'j,t}$ that is attributable to foreign TFP shocks. The exclusion restriction is that the technology shocks are uncorrelated with taste and trade cost shocks in the relevant sample and net of the fixed effects, and thus only affect the share ratios through changing the prices.

The expenditure shares on the left-hand side are sourced from WIOD. Because the WIOD expenditure shares are only available starting in 1995, the years present in the estimation sample are 1995-2007. The price changes on the right-hand side are the output price indices (`GO_P`) from KLEMS. It must be noted that the WIOD applies the proportionality assumption to construct intermediate input expenditure shares. That is, there exist actual data the total amount of sector j intermediate inputs country n imports from country m. However, from then on a proportionality assumption is applied to attribute these imported inputs to using sectors i. Thus, there is no variation in $\widehat{\pi}^x_{mj,ni,t}$ across i within the same $m \times j \times n$ cell. To avoid artificially multiplying the number of observations in the sample by a factor of J, we only use one using sector i per $m \times j \times n$ cell as the estimation sample. We cluster the standard errors at the destination-source-reference country level.

The TFP instruments are the Solow residuals constructed in a standard way from the KLEMS data. Thus, importantly, the TFP shocks used as instruments in this estimation are *not* the composite supply shocks recovered in Section 4.3.

Table A4 presents the results of estimating equations (C.1) and (C.2). Column 1 reports the OLS estimates of ρ (top panel) and ε (bottom panel). The OLS estimate of ρ is 0.94, close to a Cobb-Douglas final demand elasticity. Column 2 instruments the price changes with TFP changes. The elasticity estimate rises to 1.43. Column 3 weights by the initial expenditure shares. The coefficient estimate does not change. Columns 4-5 implement alternative weights, – average and $t-1$ expenditure shares, respectively. Column 6 uses utilization-adjusted TFP from Huo, Levchenko, and Pandalai-Nayar (2020b) as instruments instead. Column 7 uses all available countries (rather than non-G7 destinations and G7 sources only). Column 8 keeps the reference country the same in the top and bottom panels.[37] Across all robustness checks, the estimates of ρ are fairly stable, ranging between 1.4 and 1.7, and significantly larger than 1 in most cases. This difference between OLS and IV suggests measurement error in (C.1). The KP-F statistics are larger than conventional thresholds in almost all cases, although they are only 9.23 when the utilization-adjusted TFP instrument is used in Column 7.

The OLS and IV estimates of display somewhat greater consensus for ε. The OLS point estimate of ε is 0.94. The baseline IV estimates are slightly lower at 0.89. Across the same range of robustness checks as above, the range of estimates is 0.86-1.2. Such evidence for the low substitutability of intermediate inputs is consistent with the recent estimates by Atalay (2017) and Boehm, Flaaen, and Pandalai-Nayar (2019), who find even stronger complementarity, albeit with even shorter frequencies than our annual data.

We use the estimates of $\rho = 1.43$ and $\varepsilon = 0.89$ in column 3 as our baseline calibration. However, it is clear that economically and statistically the IV estimates are stable across a number of robustness checks.

C.2 Estimation of ψ

We use the responses of labor hours to shocks to estimate ψ. To do this in a theory-consistent way, we have to use the structure of the model. The labor supply in vector notation is:

$$\ln \mathbf{H}_t = \psi(\ln \mathbf{W}_t - \mathbf{\Pi}^f \ln \mathbf{P}_t),$$

Turning to the production side, the labor demand (3.8) stacked into an $NJ \times 1$ vector is:

$$\ln \mathbf{W}_t - \ln \mathbf{P}_t = \ln \mathbf{Y}_t - \ln \mathbf{H}_t.$$

This means that

$$\begin{aligned}
\ln \mathbf{H}_t &= \frac{\psi}{1+\psi}(\ln \mathbf{Y}_t + (\mathbf{I} - \mathbf{\Pi}^f \ln \mathbf{P}_t) \\
&= \frac{\psi}{1+\psi}(\ln \mathbf{Y}_t + (\mathbf{I} - \mathbf{\Pi}^f \boldsymbol{\mathcal{P}} \ln \mathbf{Y}_t) \\
&= \frac{\psi}{1+\psi}\left(\mathbf{I} + (\mathbf{I} - \mathbf{\Pi}^f \boldsymbol{\mathcal{P}}\right) \mathbf{\Lambda} \ln \mathbf{Z}_t \\
&= \boldsymbol{\mathcal{H}}(\psi, \rho, \varepsilon) \ln \mathbf{Z}_t.
\end{aligned}$$

This equation, which relates changes in equilibrium nominal wages and equilibrium prices $\ln \mathbf{W}_t - \mathbf{\Pi}^f \ln \mathbf{P}_t$ to changes in equilibrium hours $\ln \mathbf{H}_t$, can be used to estimate ψ.

The derivation of this equation makes clear that the vector of TFP shocks $\ln \mathbf{Z}_t$ act as both labor demand shifters through their effect on $\ln \mathbf{Y}_t$, but also as labor supply and demand shifters through their effects on equilibrium final goods prices $\mathbf{\Pi}^f \ln \mathbf{P}_t$ and firm intermediate goods prices $\ln \mathbf{P}_t$. This is a threat to identification, as to estimate the labor supply elasticity ψ, we need to isolate shocks that shift the labor demand curve but hold the labor supply curve approximately constant. An additional threat to identification is shocks to labor supply. Introducing an explicit labor supply shock, as in the multi-shock model in Section 5.2, labor supply

[37]For robustness, we have also used the US as the reference country for all other countries.

Table A4: Estimates of ρ and ε

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
			Alternative Weights			Varied Instruments	Varied Samples	
	OLS	IV	IV	IV	IV	IV	IV	IV
ρ	0.94	1.43	1.43	1.41	1.51	1.68	1.52	1.49
SE	(0.03)	(0.16)	(0.09)	(0.09)	(0.15)	(0.19)	(0.14)	(0.12)
Obs	152,742	115,195	114,931	115,195	115,195	114,931	414,129	114,299
First stage K-P F		245.71	90.79	94.51	82.87	9.23	161.49	69.42
FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
ε	0.94	0.86	0.89	0.96	0.97	1.10	1.15	0.89
SE	(0.02)	(0.20)	(0.17)	(0.08)	(0.34)	(0.29)	(0.12)	(0.17)
Obs	146,093	114,701	114,467	114,701	114,701	114,372	415,377	114,467
First stage KP-F		285.42	307.8	345.29	363.25	31.83	129.45	307.80
FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Weights			Baseline	Average	Lagged	Baseline	Baseline	Baseline
Instrument		Baseline	Baseline	Baseline	Baseline	Utilization-adjusted TFP	Baseline	Baseline
Sample	Baseline	Baseline	Baseline	Baseline	Baseline	Baseline	All countries	Constant Reference

Notes: Standard errors clustered at the destination-source-reference country level in parentheses. This table presents results from the OLS and IV estimation of (C.1) and (C.2). The fixed effects used in each regression are $n \times m \times m' \times t$. The instruments are the relative Solow residuals, with the Kleibergen-Papp first stage F-statistic reported except in Column 6 where they are relative utilization-adjusted TFP shocks estimated in Huo, Levchenko, and Pandalai-Nayar (2020b). The "baseline" weights in columns 3, 6, 7 and 8 are the 1995 share ratios. Column 4 uses an average share ratio as weights and column 5 uses lagged share ratios $\pi^f_{mnj,t-1}$ and $\pi^x_{mj,ni,t-1}$. All columns use the baseline sample of G7 source countries/non G7 destinations except column 7 which uses all source and destination countries. Column 8 uses a constant reference country for estimating ρ and ε.

can be written as:

$$\ln H_{nj,t} = \psi \left(\ln W_{nj,t} - \ln P_{n,t}\right) + \psi \left(1 - \xi^H_{nj,t}\right). \tag{C.3}$$

To the extent that the labor supply shocks $\xi^H_{nj,t}$ are correlated with the $\ln \mathbf{Z}_t$ that are driving the shifts in labor demand through their equilibrium impact on $\ln W_{nj,t}$, the labor supply shocks present an additional threat to identification. As a result, we cannot simply estimate ψ by regressing hours on either observed or model-implied wages. In fact, if TFP shocks in some sectors increase using sector labor demand and also decrease the final goods price index for consumers, thereby decreasing labor supply, we would expect the estimate of ψ from a regression of hours growth on wage changes to be biased towards zero.

To address this issue, we select a plausibly exogenous set of $\ln \mathbf{Z}_t$ shocks, and use them to estimate the labor supply elasticity. Our strategy is loosely inspired by the Shea (1993a,b) instruments that have been used to estimate the elasticity of industry supply curves. These papers use the patterns of input-output relationships to select subsets of sectors that are important downstream destinations for a sector's sales – thereby acting as a demand shifter — but are not themselves important sources of inputs to those downstream sectors. In that case, the sector subset can be viewed as shifting industry output demand curves without simultaneously shifting industry cost curves, permitting estimation of an output supply elasticity. We follow a similar approach. In our context we need to isolate plausibly exogenous movements in industry *labor* demand.

The set of sectors we treat as exogenous is an intersection of two sets. The first is the set of sectors least important in final consumption. TFP shocks in these sectors will have minimal impact on the final goods price index, and therefore, minimal impact on the household's labor supply to any sector. Precisely, following Shea (1993a,b), for each country we retain the 10% of all sectors with the lowest consumption shares. This limits potential endogeneity from labor supply shifts. To obtain a set of instruments that are relevant, we next isolate the set of sectors is the top 30% most important suppliers for a using sector. Shocks to these sectors will affect the input price bundle, and shift labor demand for the using sector. Selecting the intersection of these two sets picks out sectors that are not important in final consumption, but are important as input suppliers.

Intuitively, these are sectors whose shocks are likely to be the pure labor demand shocks for the using sector: productivity improvements in a sector's intermediate input suppliers will increase labor demand in those sectors. At the same time, because these sectors are not important in final consumption, shocks to those sectors are not shifting labor supply to the using sector. There remains the possibility that the exogenous (admitted) shocks are correlated with the potentially endogenous (excluded) shocks, and other labor supply shocks. We use the sectoral Solow residuals as the shocks $\ln \mathbf{Z}_t$. In our earlier work (Huo, Levchenko, and Pandalai-Nayar, 2020b), we found that the Solow residuals have at most a modest correlation across country-sectors. Thus, the correlation between the residuals and the admitted exogenous shocks is likely minimal. To clean out any remaining common component from these shocks, we take out the time fixed effects from the panel of shocks.[38]

Note that the matrix $\boldsymbol{\mathcal{H}}(\psi, \rho, \varepsilon)$ that relates shocks to hours responses is a function of ρ and ε. In addition, $\boldsymbol{\mathcal{H}}(\psi, \rho, \varepsilon)$ is a nonlinear function ψ, and thus we use the Non-Linear Least Squares estimator:

$$\hat{\psi} = argmin_{\psi} \left(\ln \mathbf{H}_t - \left(\boldsymbol{\Lambda}^H (\psi, \rho, \varepsilon) \cdot \mathbf{S} \right) \ln \mathbf{Z}_t \right)^2 , \tag{C.4}$$

where $\mathbf{S}$ is the selection matrix, whose entries are either 0's or 1's, that picks out for each destination country-sector (n, j) the exogenous set of sectors, as described above.

The maintained identifying assumption is that *each* of the Solow residuals in the exogenous set of sectors for each sector (n, j) is orthogonal to the error term in sector (n, j). While in principle we could introduce a separate NLS estimating equation for each exogenous sector and using sector and estimate a range of ψ's using exclusively time variation, this is both very cumbersome and likely to be very noisy as the time series is relatively short. We gain statistical power by pooling all equations together and estimating a single ψ that minimizes (C.4).

This requires that the transformed *sum* of the Solow residuals in the exogenous set of sectors is orthogonal to the error term for each destination sector. Of course, the maintained identifying assumption that each excluded sector is orthogonal is a sufficient albeit not necessary condition for their sum to be orthogonal. We favor this approach as (C.4) is the most parsimonious condition that allows us to estimate ψ on the pooled dataset of countries, sectors, and years.

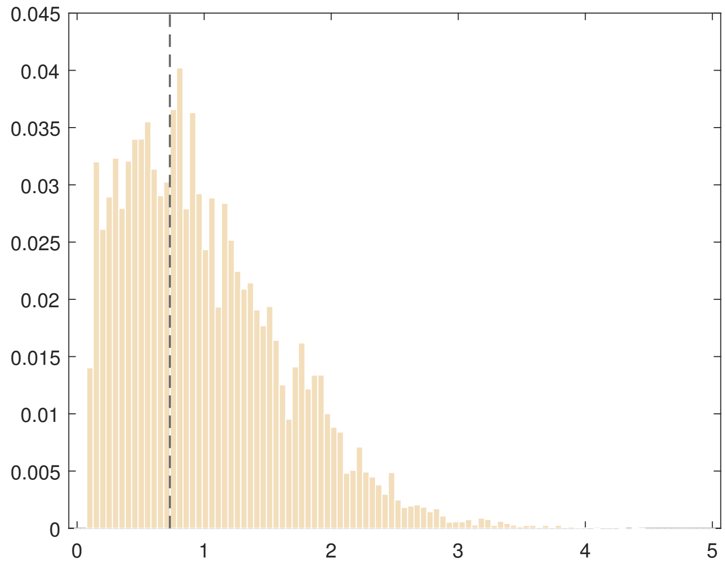
Our baseline estimate of ψ using this procedure is 0.723. Note that it is very close to the Frisch elasticity suggested by Chetty et al. (2011) as appropriate for macro models. This is reassuring, as our sectoral data and quantitative setting are not the ideal setting to estimate the labor supply elasticity due to a number of measurement challenges. In particular the composition of hours worked is likely to vary importantly over the cycle (e.g. Solon, Barsky, and Parker, 1994). Compounding the problem, the nonlinear relationship between the Frisch elasticity and the hours' response to shocks makes it challenging to estimate ψ. As we acknowledge the limitations of our setting for estimating the Frisch elasticity, we also report results when using the value of $\psi = 0.75$ suggested Chetty et al. (2011), treating it as an externally calibrated parameter.

The NLS estimator is also a function of the elasticities ρ and ε estimated above. To take into account how the uncertainty over ρ and ε translates into uncertainty over ψ, we draw 1000 times from the distributions of ρ and ε estimated above, and estimate ψ under each of these pairs of ρ and ε.[39] Taking the distribution of estimates of ψ across the draws of ρ and ε gives us the empirical marginal distribution of ψ. Figure A5 displays the histogram of estimates of ψ. As ρ and ε vary, the resulting distribution of ψ is centered around the baseline value, but with a thicker upper tail.

[38]We have also assessed whether the admitted shocks, after fixed effects, are correlated with the labor shocks we back out in Section 5.2. We found no correlation.

[39]Note, we assume that the distributions of ρ and ε are independent. We have experimented with estimating (C.1) and (C.2) by Seemingly Unrelated Regressions (SURs). The results suggested close to zero covariance between the estimates, implying our assumption of independence is reasonable. We do not use these estimates as our baseline, as existing econometric tools to estimate SURs with separate instruments do not permit clustering of standard errors, and thus in our setting produce artificially low standard errors.

Figure A5: The distribution of ψ



Notes: This figure displays the histogram of the values of ψ estimated by (C.4), drawing 1000 times from the distributions of ρ and ε implied by estimates in column 3 of Table A4. The vertical dashed line is the baseline point estimate of ψ.

C.3 Estimation of Shock Processes

The estimation sample is the G7 countries, for which we have the longest time series of shocks. As discussed in Section 4, while estimating an unrestricted process for shocks is not possible due to the short panel of measured shocks and the large number of parameters to be estimated, we impose minimal restrictions that allow the shocks to be correlated (as the measured shocks are), and further, allow for spillovers between country-sectors.

Equations (4.2)-(5.4) state the estimating equations for the shock processes in the one- and four-shock models respectively. In particular, our specification allows for contemporaneous shock correlations between country-sectors, but restricts the structure of lagged spillovers. We permit a country-sector specific lagged autoregressive parameter, so country-sector shocks can be persistent. We restrict lagged spillovers to be common within a country (across sectors), and zero otherwise. We allow for a full variance-covariance matrix of the error terms, which amounts to assuming completely unrestricted contemporaneous spillovers. The sample variance-covariance matrix of the residuals serves as the estimate of the covariance matrix of the shock innovations.

The choice of restrictions strikes a balance between relative parsimony, which improves the precision of the parameters estimated, and sufficient flexibility to replicate the measured shock correlations in the data. We experimented with other processes using methods such as LASSO regressions to estimate the process for the intertemporal shocks without much change to the simulated shock correlations. In particular, we have modified the estimating equations to also include a sector-specific lagged spillover term, but these coefficients were all insignificant, and so we use the more parsimonious process in the baseline analysis. Table A5 summarizes the estimation results.

Table A5: Shock Processes: Autoregressive and Spillover Parameters

	Mean	Median	25th pctile	75th pctile
One-Shock Model				
		$\ln Z_{nj,t}$		
Own lag (ρ_{nj})	0.862	0.880	0.831	0.886
Spillover lag (δ_n)	-0.001	-0.002	-0.002	0.000
Four-Shock Model				
		$\ln \xi^Z_{nj,t}$		
Own lag (ρ_{nj})	0.877	0.875	0.862	0.894
Spillover lag (δ_n)	-0.002	-0.002	-0.002	-0.001
		$\ln \xi^X_{nj,t}$		
Own lag (ρ_{nj})	0.852	0.848	0.837	0.869
Spillover lag (δ_n)	0.002	0.002	0.001	0.002
		$\ln \xi^H_{nj,t}$		
Own lag (ρ_{nj})	0.901	0.904	0.888	0.917
Spillover lag (δ_n)	-0.001	-0.001	-0.002	0.000
		$\ln \xi^I_{nj,t}$		
Own lag (ρ_{nj})	0.930	0.933	0.929	0.935
Spillover lag (δ_n)	-0.005	-0.005	-0.005	-0.005

Notes: This table presents results from estimating the shock stochastic processes (4.2)-(5.4). The "One-Shock Model" panel presents the results for the composite shock estimated and used in Section 4. The "Four-Shock Model" panel presents the results for the 4 shocks estimated and used in Section 5.2. The measures are summary statistics of the coefficients in the sample of sectors and countries.

D. Sensitivity and Additional Exercises

D.1 Sensitivity

This subsection reports various sensitivity exercises for the network model results in Sections 3-4.

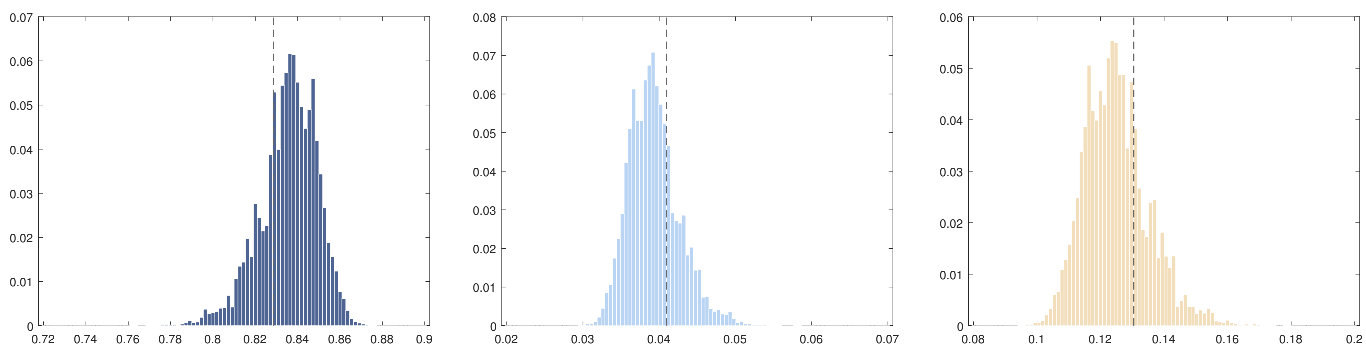
Table A6: GDP Growth and HP-Filtered GDP Correlation in G7 Countries

	Mean	Median	25th pctile	75th pctile	St.dev. (GDP)
GDP growth rate	0.358	0.333	0.122	0.552	1.667
HP-Filtered GDP	0.395	0.460	0.040	0.585	1.736

Notes: This table presents the summary statistics of the correlations of GDP growth rates and HP-filtered GDP in the sample of G7 countries. We use $\lambda = 100$ for the HP-filter parameter, as the data are annual.

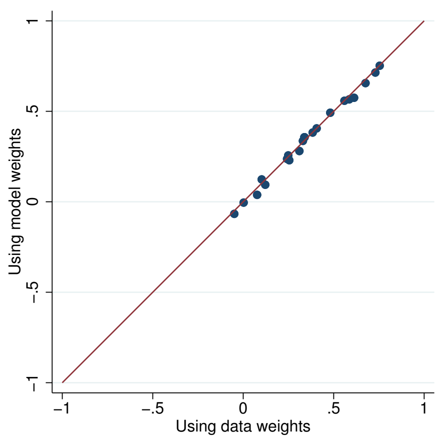
Figure A6: Distribution of Direct Effects versus Transmission

(a) shock correlation (b) bilateral transmission (c) multilateral transmission

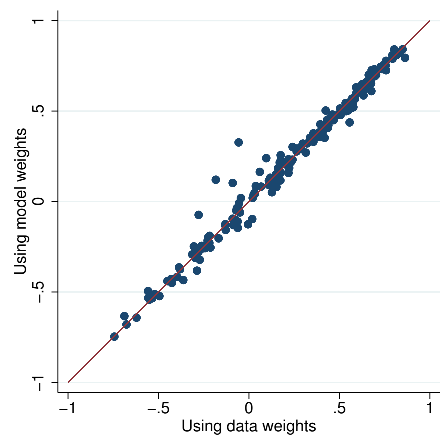


Notes: This figure displays the distribution of relative contribution of shock correlation, bilateral transmission and multilateral transmission to international comovements in the G7 countries. The elasticities of substitution ρ and ε are drawn from normal distributions and the Frisch elasticity ψ is fixed at 0.72.

Figure A7: GDP Correlations Under Data and Model Domar Weights



G7



All Countries

Notes: This figure displays a scatter plot of the GDP growth correlation, when GDP growth is computed by aggregating sectoral growth using the data Domar weights against the model Domar weights.

D.2 The Role of Individual Sectors in Comovement

We next address the question of whether some sectors systematically contribute more to aggregate comovement than others. Equation (2.2) can be written as $d\ln G_{n,t} = \sum_i d\ln \mathcal{V}_{ni,t}$, where $\mathcal{V}_{ni,t} \equiv \sum_{k=0}^{\infty} \sum_{\ell} s_{\ell ni,k}\theta_{\ell i,t-k}$ is the contribution of shocks in sector i anywhere in the world to GDP growth of country n (not to be confused with value added in sector i country n). The GDP covariance is simply additive in the covariances of sectoral $d\ln \mathcal{V}_{ni}$ with foreign GDP:

$$\rho_{nm} = \sum_i \frac{\mathrm{Cov}(d\ln \mathcal{V}_{ni}, d\ln G_m)}{\sigma_n \sigma_m}. \tag{D.1}$$

Thus, we can decompose the overall GDP correlation into components due to shocks in individual sectors. Further, we can treat $d\ln \mathcal{V}_{ni}$ as its own "economy," and decompose its covariance with foreign GDP into shock correlation and transmission:

$$d\ln \mathcal{V}_{ni,t} = \underbrace{\sum_{k=0}^{\infty} s_{nni,k}\theta_{ni,t-k}}_{\mathcal{D}_{ni}} + \underbrace{\sum_{k=0}^{\infty} s_{mni,k}\theta_{mi,t-k}}_{\mathcal{P}_{ni}} + \underbrace{\sum_{k=0}^{\infty} \sum_{n' \neq n,m} s_{n'ni,k}\theta_{n'i,k}}_{\mathcal{T}_{ni}}. \tag{D.2}$$

$$\begin{aligned} \mathrm{Cov}(d\ln \mathcal{V}_{ni}, d\ln G_m) &= \underbrace{\mathrm{Cov}(\mathcal{D}_{ni}, \mathcal{D}_m)}_{\text{Shock Correlation due to } i} \\ &+ \underbrace{\mathrm{Cov}(\mathcal{D}_{ni}, \mathcal{P}_m) + \mathrm{Cov}(\mathcal{P}_{ni}, \mathcal{D}_m) + \mathrm{Cov}(\mathcal{P}_{ni}, \mathcal{P}_m)}_{\text{Bilateral Transmission due to } i} \\ &+ \underbrace{\mathrm{Cov}(\mathcal{D}_{ni} + \mathcal{P}_n + \mathcal{T}_{ni}, \mathcal{T}_m) + \mathrm{Cov}(\mathcal{T}_{ni}, \mathcal{D}_m + \mathcal{P}_m)}_{\text{Multilateral Transmission due to } i}. \end{aligned} \tag{D.3}$$

This way, we can evaluate whether the contribution of sector i to GDP comovement is due primarily to sector i shocks being correlated across countries, or to the fact that shocks to i transmit across countries.

Figure A8 plots the results of this approach in our baseline model, averaging across countries for each sector. We express the terms in equation (D.3) relative to the aggregate correlation between countries n and m to interpret the results as shares. There is substantial heterogeneity in the average contributions of individual sectors to GDP comovement. Services sectors tend to be the most prominent, mirroring their relatively large size. The Wholesale and Retail Trade sectors are among the most important. Sectors like Financial Intermediation, Health and Social Work, and Transport Equipment contribute negatively to aggregate comovement. Even more so than our aggregate findings, the shock correlation term is predominant at sector level, and transmission is a small minority.

D.3 Correlation Structure of Shocks

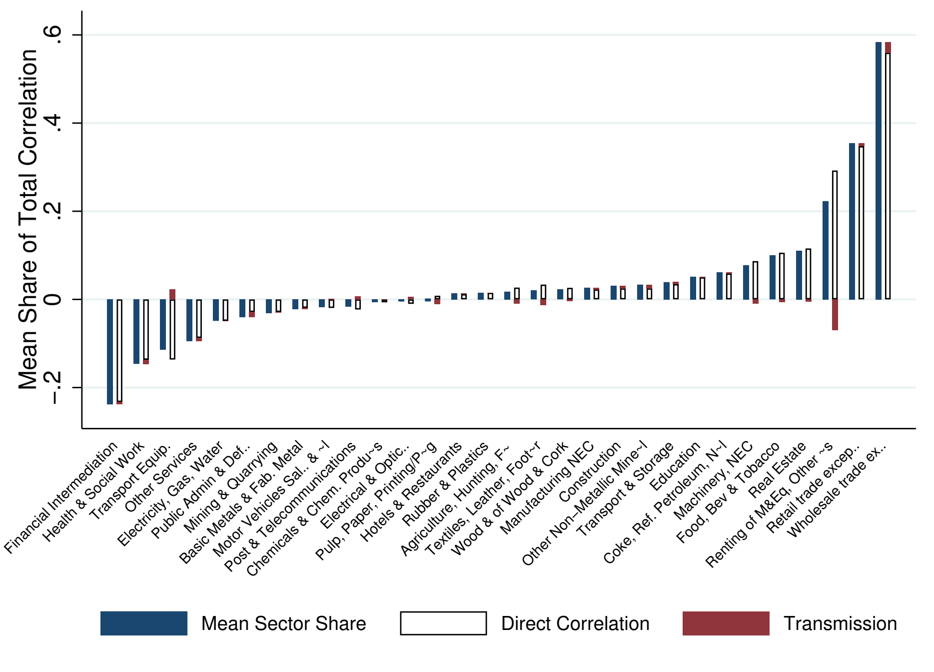
We now investigate whether the recovered shocks have a factor structure. We begin by estimating a factor model featuring a global factor F_t^g, a country factor $F_t^m, \forall m \in N$ and a sector factor $F_t^i, \forall i \in J$ on the composite shocks $dZ_{mi,t}$ in the one-shock model:

$$\ln Z_{mi,t} = \gamma_{mi}^g F_t^g + \gamma_{mi}^i F_t^i + \gamma_{mi}^m F_t^m + u_{mi,t},$$

where γ^g, γ^i and γ^m are the corresponding global, sector and country loadings. The assumption is that $\gamma_{mk}^i = 0, k \neq i$, and $\gamma_{ni}^m = 0, n \neq m$. We then compute variance decompositions of the shocks into components attributable to each type of factor. Note that the factor estimation does not impose that the factors are orthogonal, however in practice we found that the estimated factors were uncorrelated, making the variance decomposition straightforward.

Table A7 displays the mean (p25, p75) share of the variance attributable to each type of factor for the composite shock, as well as the four shocks extracted in the business cycle accounting extension in Section

Figure A8: Mean Sectoral Shares in Aggregate GDP Correlations, G7, 1978-2007



Notes: This figure displays the average share of total bilateral correlation accounted for by each sector (blue bars), as well as the decomposition in equation (D.3) into the component due to the shock correlation (white bars) and bilateral+multilateral transmission (red bars), for G7 country pairs over the period 1978-2007.

5.2. Clearly, the common global factor does not explain much of the variance of the shocks, with the partial exception of the investment shock. A larger role is played by the country factors, and, to a lesser extent, sectoral factors. Overall, however, there is no clear dominant type of factor explaining the bulk of the variation in the extracted shocks. For all shocks except the investment shock, the three sets of factors on average explain less than 50% of the variance of the series. For the investment shocks, the three factors explain over 90% of the variation on average.

The second exercise we conduct is to regress the panel for each type of shock on country-time and sector-time fixed effects and compute the partial R^2 due to each type of fixed effects. This captures the variation absorbed by the country-time specific components and sector-time specific means. The results in Table A8 illustrate a similar pattern as with the estimated factors: with the exception of the investment shock, no fixed effects explain a majority of the variation for any shock. The sector-time effects play a somewhat larger role. For the investment shock, country-time effects are by far the dominant component.

In sum, our results suggest that common global factors are not the driving source of shock correlation, with the partial exception of the investment shock. Rather, common country-specific and (to a lesser extent) sector-specific components help generate the observed correlation patterns. However, a substantial fraction of variation in the shocks is idiosyncratic.

Table A9 reports the sector-specific correlations of shocks. The first column averages the correlations of shocks within the same sector (for example, correlation of the textile sector in the US with the textile sector in the UK). The second column is the average of the cross-border correlations of different sectors (for example, correlation of the textile sector in the US with the machinery sector in the UK). There is indeed quite a bit of variation across sectors in these averages. The mean within-sector correlation is 0.11, but the range is between −0.01 and 0.376. Within-sector correlations tend to be higher than cross-sector correlations, which are lower on average and exhibit less range.

Table A7: Factor Decomposition of Shocks

Factor	$\ln Z_{mi,t}$	$\ln \xi^{Z}_{nj,t}$	Shock $\ln \xi^{H}_{nj,t}$	$\ln \xi^{X}_{nj,t}$	$\ln \xi^{I}_{nj,t}$
Global	0.13	0.11	0.11	0.08	0.27
(p25, p75)	(0.01, 0.20)	(0.02, 0.16)	(0.01, 0.17)	(0.01, 0.12)	(0.23, 0.33)
Country	0.20	0.17	0.21	0.18	0.52
(p25, p75)	(0.05, 0.30)	(0.03, 0.25)	(0.04, 0.33)	(0.04, 0.29)	(0.46, 0.59)
Sector	0.13	0.13	0.13	0.12	0.10
(p25, p75)	(0.02, 0.22)	(0.02, 0.20)	(0.02, 0.18)	(0.02, 0.19)	(0.03, 0.16)

Note: This table reports the summary statistics for the shares of the variances accounted for by the factor in the row, for each type of shock in the column. Sample of 30 sectors for G7 countries from 1978-2007 as described in the text.

Table A8: Explanatory power of country-time and sector-time effects

Fixed Effects	$\ln Z_{mi,t}$	$\ln \xi^{Z}_{nj,t}$	Shock $\ln \xi^{H}_{nj,t}$	$\ln \xi^{X}_{nj,t}$	$\ln \xi^{I}_{nj,t}$
Country-time	0.11	0.11	0.17	0.07	0.85
Sector-time	0.22	0.20	0.22	0.20	0.23

Note: This table presents the partial-R^2 of country-time and sector-time effects in a regression of a panel of a shock series on country-time and sector-time effects. Each column presents the results for one of the shock series considered in the paper. The sample is 30 sectors for the G7 countries between 1978-2007 as described in the text.

D.4 The Role of The Input Network

Our next counterfactual to understand the role of the input network studies whether trade in intermediate inputs and final goods amplifies GDP comovement. To assess this, we compute versions of the model in which trade in inputs and trade in final goods, respectively, are shut down. These scenarios are constructed as subsets of the full autarky scenario in Section 4.6.2, by reallocating only intermediate or final shares to domestic sources. Figure A9 presents the change in average GDP comovement for the G7 and all countries in these counterfactual scenarios.

Overall, both intermediate and final trade increase comovement on average. The impact of intermediate input trade on synchronization is more than twice as large as the impact of final trade, on average. This is sensible, as input trade is governed by the lower substitution elasticity and thus transmission is stronger. The main results of the full autarky experiment in Section 4.6.2 carry over to these more limited changes in trade costs. In particular, the Δ Shock Correlation term is consistently negative in both scenarios.

D.5 The Trade-Comovement Relation

Table A10 reports the results of running the "standard" trade-comovement regression in our data and the network model. This is a regression of bilateral real GDP correlation on a measure of bilateral trade intensity. A long literature following Frankel and Rose (1998) tries to understand why economies that trade more display higher GDP comovement in the data. Input linkages have been suggested as an explanation for the trade-

Table A9: Sector-Specific Shock Correlations

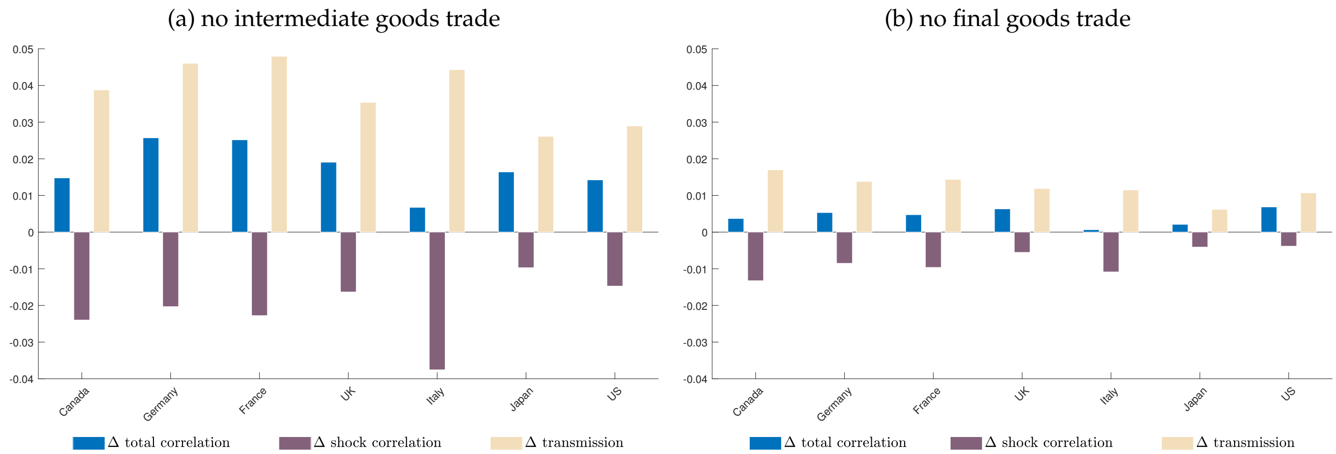
	Sector	Mean $\varrho^{Z}_{mj,nj}$	Mean $\varrho^{Z}_{mi,nj}$, $i \neq j$
AtB	agriculture hunting forestry and fishing	0.103	0.008
C	mining and quarrying	-0.013	0.027
D15t16	food beverages and tobacco	0.001	0.018
D17t19	textiles textile leather and footwear	0.169	0.049
D20	wood and of wood and cork	0.037	0.048
D21t22	pulp paper paper printing and publishing	0.195	0.069
D23	coke refined petroleum and nuclear fuel	-0.003	0.024
D24	chemicals and chemical products	0.168	0.054
D25	rubber and plastics	0.192	0.057
D26	other nonmetallic mineral	0.290	0.077
D27t28	basic metals and fabricated metal	0.196	0.063
D29	machinery nec	0.376	0.075
D30t33	electrical and optical equipment	0.357	0.083
D34t35	transport equipment	0.116	0.042
D36t37	manufacturing nec; recycling	0.053	0.054
E	electricity gas and water supply	0.173	0.026
F	construction	0.087	0.044
G50	sale maintenance and repair of motor vehicles	0.044	0.046
G51	wholesale trade and commission trade	0.064	0.049
G52	retail trade except of motor vehicles	0.003	0.034
H	hotels and restaurants	0.026	0.014
I60t63	transport and storage	0.212	0.072
I64	post and telecommunications	0.219	0.021
J	financial intermediation	0.034	0.023
K70	real estate activities	0.061	0.049
K71t74	renting of m&eq and other business activities	0.105	0.047
L	public admin and defense; compulsory social security	0.086	-0.016
M	education	0.050	-0.027
N	health and social work	0.031	-0.044
O	other community social and personal services	0.049	0.013
	mean	0.110	0.035
	min.	-0.013	-0.044
	max.	0.376	0.083

Note: This table reports the average correlations of shocks across countries. The first column reports the same-sector correlations. The second column reports the cross-sector correlations.

comovement puzzle in a number of papers (see for instance Kose and Yi, 2006; di Giovanni and Levchenko, 2010; Johnson, 2014). Quantitatively, however, models have trouble generating even the same order of magnitude as the empirical relationship (model coefficients are often <10% of their empirical counterparts).

When it comes to GDP correlations, our model matches perfectly the trade-comovement relationship found in the data, by virtue of matching GDP growth rates for each country-year. Columns (1)-(2) run the trade-comovement regression in the data, while columns (3)-(4) do the same in the model. Since each GDP correlation can be additively decomposed into the Shock Correlation and Transmission components, we can also run the

Figure A9: Change in GDP Correlation between Trade and No Trade Models: Decomposition



Notes: Panel (a) displays the mean change in the GDP correlation going from no intermediate goods trade to observed trade (blue bars), decomposing it into the mean Δshock correlation$_{mn}$ (brown bars) and transmission terms (yellow bars). Panel (b) displays the mean change in the GDP correlation going from no final goods trade to observed trade (blue bars), decomposing it into the mean Δshock correlation$_{mn}$ (brown bars) and transmission terms (yellow bars).

trade comovement regressions with those as the dependent variables. This is done in columns (5)-(8). We implement this exercise on the model with 29 countries in Section 5.1 to increase the sample size. It turns out that trade intensity is correlated with both components of total GDP comovement. However, the bulk of the overall slope (0.73 out of 0.85) is accounted for by the positive relationship between trade intensity and shock correlation. This underscores the relevance of the Imbs (2004) critique of trade-comovement regressions: bilateral trade intensity can be a proxy of country similarity, and thus of correlated shocks.

Table A10: The Trade-Comovement Relation

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Data		Model					
Dep. Var:	Bilateral GDP growth correlation				Shock correlation		Transmission	
Trade intensity (avg)	0.085*** (0.012)							
Trade intensity (1995)		0.086*** (0.011)						
Trade intensity (model, avg)			0.085*** (0.011)		0.073*** (0.011)		0.012*** (0.001)	
Trade intensity (model, 1995)				0.085*** (0.011)		0.073*** (0.011)		0.012*** (0.001)
N	406	406	406	406	406	406	406	406

Notes: This table presents the results of a regression of bilateral GDP growth correlation on trade intensity for the data and in the 29-country model in Section 5.1. Trade intensity is defined as the sum of bilateral flows over the sum of the two countries' GDPs. The first row uses the average trade intensity over the 1995-2007 period, while the second row uses the initial intensity.

D.6 Role of Financial Integration

This subsection explores an alternative international financial market structure. Whereas in the baseline countries are in financial autarky, we now extend the model to allow countries to trade a complete set of state-contingent securities. In this setting the goods prices also hinge on the financial flows across countries and the channels of international transmission are different.

We make the following modifications relative to the baseline economy. First, we allow the period-utility to be non-GHH:

$$U\left(C_{n,t}, H_{n1,t}, \ldots, H_{nJ,t}\right) = \frac{C_{n,t}^{1-\sigma}}{1-\sigma} - \sum_j H_{nj,t}^{1+\frac{1}{\psi}},$$

where σ controls the wealth effects of international transfers. The flexibility to accommodate a wealth effect of flexible magnitude is important because the impact of the international financial transfers depends on it. Second, the budget constraint for consumers in country n is

$$\sum_t \sum_{s^t} P_{n,t}(s^t) C_{n,t}(s^t) = \sum_t \sum_{s^t} \left(\sum_j W_{nj,t}(s^t) H_{nj,t}(s^t) + \sum_j R_{nj,t}(s^t) K_{nj} \right),$$

where s^t indexes histories of shocks up to time t. For simplicity we assume that the capital stock is fixed and focus on the impact response to shocks.

With complete markets, in all states, the marginal utility of 1 unit of nominal output across countries will be equalized, and the relative consumption and the real exchange rate move in the opposite directions (Backus and Smith, 1993):

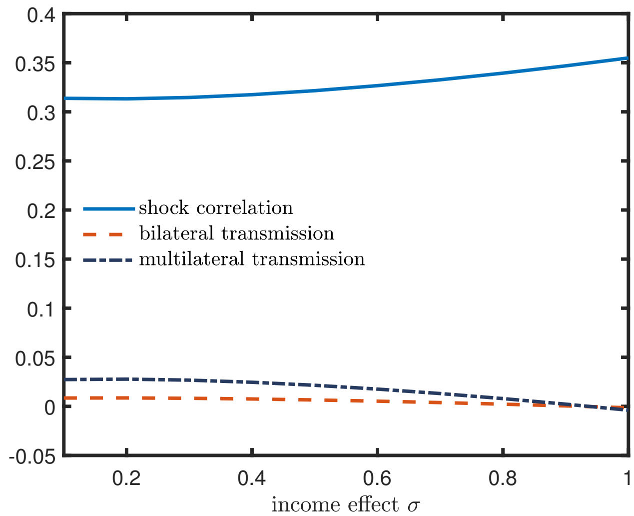
$$-\sigma \ln C_{n,t} - \ln P_{n,t} = -\sigma \ln C_{m,t} - \ln P_{m,t}.$$

However, the insurance channel on the final goods consumption does not necessarily result in a stronger international transmission to measured real GDP. In fact, with complete markets, a positive TFP shock in country n directly induces an increase in country m's consumption, but it tends to lower country m's labor supply. At the same time, the magnitude the terms of trade response is dampened under complete markets as discussed in Heathcote and Perri (2002), which further lowers foreign firms' incentive to expand production capacity. These forces weaken the transmission of shocks from one country's real GDP to another. Figure A9 shows how the international comovement decomposition varies with σ. As expected, the shock correlations appear to be more important than our baseline economy. In the special case with $\sigma = 1$ and unitary elasticities of substitution, the relative goods prices remain constant in response to shocks, and the shock correlation will account for 100% of the international comovement, as we show analytically in Appendix B.1.

It is worth noting that the above results are only about how international financial integration in the form of complete markets affects the transmission of the real supply shocks that we focus on in this paper. They do not imply that global capital flows are not important in shaping international business cycle comovement. International financial integration in combination with imperfect asset markets can make room for shocks to the financial system that could be important sources of international comovement (Jordà et al., 2019; Miranda-Agrippino and Rey, 2020; Boehm and Kroner, 2023).

Figure A9: Correlated Shocks and Transmission with Financial Integration

(a) role of income effects



Notes: This figure displays the average contributions of correlated shocks and transmission with different strengths of income effects in complete markets.

D.7 Additional Exercises in the Four-Shock Model

Correlates of Recovered Shocks. We stress that the 4 shocks recovered in Section 5.2 use no external information or exogenous variation. Thus, they are consistent with a variety of microfoundations. To understand whether these recovered shocks correspond closely to existing independently identified shocks, we collect a number of shocks from earlier studies, and correlate them with each of our 4 recovered shocks. We make use of the following standard shock series: (i) the Fernald (2014) utilization-adjusted TFP; (ii) the Barsky and Sims (2011) news of future TFP; (iii) the Levchenko and Pandalai-Nayar (2020) sentiment shocks; (iv) the Romer and Romer (2004) monetary policy shocks, updated by Coibion et al. (2017); (v) federal spending (Ramey, 2011) and tax (Romer and Romer, 2010) shocks; (vi) oil price increases (Hamilton, 2003) and identified oil supply shocks (Baumeister and Hamilton, 2019); (vii) financial shocks proxied by excess bond premia (Gilchrist and Zakrajšek, 2012), and (viii) uncertainty shocks in the form of innovations to VIX (Bloom, 2009) and to the Baker-Bloom-Davis index of policy uncertainty. Many of these shocks come at monthly or quarterly frequency, and we convert them to annual frequency to match with our data.

Unfortunately, most of these identified shocks are specific to the US. The ideal exercise here would collect all possible independently identified shocks for all countries to determine which shocks are the most promising for explaining the patterns of comovement. To our knowledge, however, collections of these shocks do not exist for multiple countries. Thus, we can only compare these identified shocks to the annual series of shocks for the US recovered from our model. We have 30 years of observations. While this exercise will not speak directly to sources of cross-country comovement, it can at least tell us whether our recovered shocks correlate closely with externally identified shocks for one country.

We begin with regressing each model-recovered shock on an individual category of identified shocks. Table A11 reports the resulting R^2's. For TFP and labor market shocks, the Barsky-Sims news shocks have the highest bivariate explanatory power. Least explainable is the intermediate input shock, with the lowest overall R^2 and no set of identified shocks having an R^2 of over 0.06. By contrast, the investment shock appears to be the most correlated with other identified shocks, with sentiment, oil, financial, and uncertainty shocks having an R^2 of 15% or more.

Of course, the externally identified shocks themselves can be mutually correlated. To see which identified shocks have the strongest conditional correlations with our recovered shocks, we regress the recovered shocks on all the identified shocks together. Note that with only 30 annual observations and 10 regressors, there are

Table A11: Projecting Recovered Shocks on Identified Shocks

	TFP		Labor		Intermediate		Investment	
	R^2	Sig.	R^2	Sig.	R^2	Sig.	R^2	Sig.
Fernald TFP	0.09		0.00		0.01		0.01	
News	0.21	*	0.14	**	0.01		0.04	*
Sentiment	0.06		0.13		0.05	*	0.18	**
Monetary	0.06		0.04		0.05	**	0.06	
Fiscal	0.02		0.06	*	0.06		0.01	
Oil	0.06		0.03		0.01		0.15	***
Financial	0.03		0.11		0.02		0.25	
Uncertainty	0.20	*	0.13		0.10		0.20	
All together	0.54		0.51		0.32		0.58	

Notes: The columns labeled "R^2" report, for each recovered shock in the column, the R^2 from a bivariate regression of the growth in that shock on identified shocks in the row. The row "All together" reports the R^2 of regressing the shock in the column on all the externally identified shocks together. The column labeled "Sig." reports the level of significance of the category of identified shocks when all the identified shocks are used as regressors together in a multivariate regression. ***: significant at the 1% level; **: significant at the 5% level; *: significant at the 10% level. Variable definitions and sources are described in detail in the text.

not that many degrees of freedom left. Nonetheless, columns labeled "Sig." report the level of significance of individual shocks when all are included in the same regression. There is some variation in which shocks are most important. Overall, news and sentiment shocks appear most correlated with our shocks, but their relative importance also varies across shocks. Fiscal, monetary, oil, and uncertainty shocks appear with varying levels of significance for individual series.

All in all, there is no clear pattern of correlation, whereby a recovered shock can be convincingly attributed predominantly to a particular externally identified shock. Nonetheless, this exercise suggests that at least for the US, our shocks bear some resemblance to prominent identified sources of business cycle fluctuations.

Historical Narrative. Unlike with recessions and expansions, of which there are typically several per country, average comovement does not exhibit large swings in our sample. There is a partial exception starting in the 90s, that we exploit to provide a historical narrative.

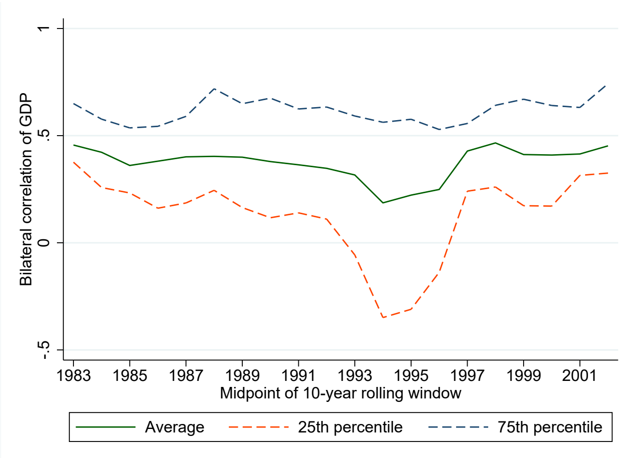
The left panel of Figure A10 plots the rolling 10-year average GDP correlation among the G7 countries over our sample period.[40] There is a dip in the mid-1990s, especially evident in the lower quartile. Digging into the country dimension, it turns out that the drop in the GDP correlation is driven by the decoupling of Japan from the rest of the G7 starting in about 1990. The right panel of Figure A10 plots the average rolling correlations for country pairs involving Japan and not involving Japan separately. The entire dip in the average correlations in the mid-1990s is driven by the fall in the correlations between Japan the the rest.

To investigate this further, Figure A11 plots the rolling 10-year correlation of GDP conditional on one shock at a time for Japan and non-Japan country pairs separately. It is clear that the most pronounced drop in the GDP correlations comes from the investment shock in particular. The pattern with the GDP coming from other shocks is less consistent. The labor shock-driven GDP also experiences a dip in correlation, but it is much milder and is mirrored by other countries' GDP correlations as well. The timing of the changes in the correlations driven by TFP and intermediate input shocks does not track the actual GDP correlation pattern. These findings are indeed consistent with the historical record. The Japanese stock market peaked on the last trading day of 1989, and proceeded to lose nearly 50% of its value over the course of 1990. That triggered the period known as "the Lost two Decades" in Japan. The stock market crash led to a recession in 1991-1993. This was followed by an even deeper recession in 1997-1998, that the rest of the G7 entirely missed. In 1997-98 Japan also experienced a banking crisis that took many years to resolve. (See Ito and Hoshi, 2020, ch. 14, for a narrative of the Japanese macroeconomy over this period.) The widespread "zombie banks lending to zombie firms" phenomenon led

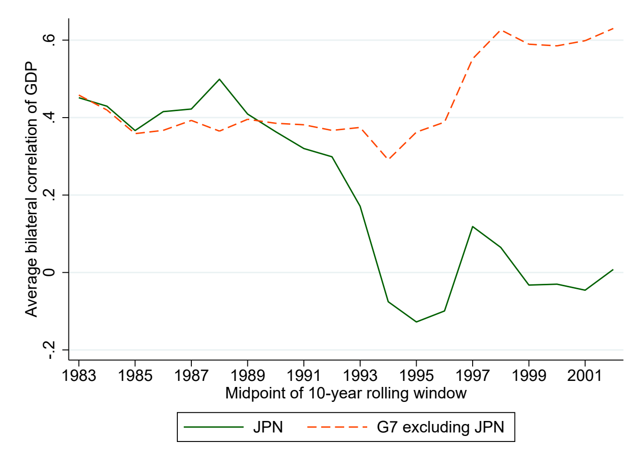
[40]This average correlation shows no discernible long-run trend between 1978 and 2007. Our follow-up work (Bonadio et al., 2021b) is dedicated to understanding this lack of trend in spite of the large increase in trade openness over this period.

Figure A10: Average Rolling GDP Correlations, G7

(a) All G7 Pairs



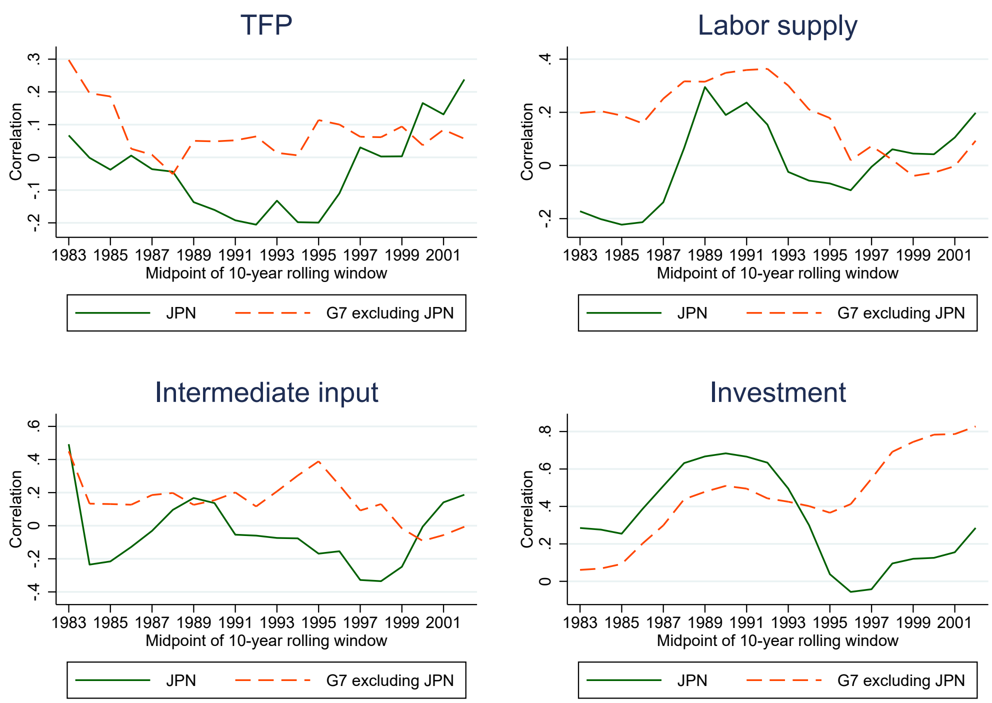
(b) Japan and Non-Japan Pairs



Notes: Panel (a) displays the average rolling 10-year GDP correlations and the 25th and 75th percentile rolling 10-year GDP correlations for all G7 country pairs. Panel (b) displays the average rolling 10-year GDP correlations separately for country pairs involving Japan and not involving Japan.

to a long-running malaise in the banking sector, characterized by under-capitalization and anemic lending to productive projects (Hoshi and Kashyap, 2004). Finally, in early 1999 Japan entered a zero-lower-bound period, alone at that time among the countries in our sample. All in all, this historical narrative is consistent with both Japan's overall decoupling from the rest of the G7 starting in the early 1990s, and with the investment shock being the primary proximate driver of that decoupling.

Figure A11: Rolling GDP Correlations, Conditional on One Shock, G7



Notes: This figure displays the average rolling 10-year GDP correlations conditional on one shock at a time. Solid lines display country pairs involving Japan, dashed lines display all other G7 country pairs.